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**ERROR ANALYSIS
FOR APOLLO EARTH PARKING ORBITS
USING RANGE RATE
AND ANGLE MEASUREMENTS**

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JANUARY 1965

NASA

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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SUMMARY

21662

Parametric studies are made of RMS errors in position and velocity for an Apollo Earth Parking Orbit (launch azimuth 90°, height 200 km), using uncertainties in range rate of ± 0.1 m/s and ± 0.03 m/s; and in angles of $\pm 2 \times 10^{-3}$ radians, $\pm 6 \times 10^{-4}$ radians, and $\pm 2 \times 10^{-4}$ radians.

Range rate and angles are measured every second by a tracking ship located at 26° 40' N and 49° 20' W in order to provide tracking during the time the spacecraft is inserted into the parking orbit. In order to compare the RMS errors in the state vector, two separate sets of error analyses are made which include and exclude the effects of an uncertainty in the tracking ship's location. For this study a total uncertainty in the tracking ship's location of approximately ± 450 meters was assumed.

It was found that when range rate and angles are measured, the angles are the dominant influence; i. e., that the position and velocity errors are approximately the same whether $\delta r = \pm 0.1$ m/s or ± 0.03 m/s. For example, with optimum tracking and an angular uncertainty of $\pm 2 \times 10^{-3}$ radians, the RMS error in position is 144.2 meters when $\delta r = \pm 0.1$ m/s and 143.8 meters when $\delta r = \pm 0.03$ m/s. Corresponding velocity errors are 0.821 m/s and 0.819 m/s. When the angular uncertainty is $\pm 2 \times 10^{-4}$ radians, the position errors are 16.6 meters and 14.7 meters and the velocity errors are 0.097 m/s and 0.084 m/s for the respective uncertainties in range rate.

However, when station location errors are included, these become the dominant influence and even with a minimum amount of tracking, the RMS errors in position approach the level of the station location error.

J. W. Johnson

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ERROR ANALYSIS FOR APOLLO EARTH PARKING ORBITS USING RANGE RATE AND ANGLE MEASUREMENTS

by
A. Marlow

The purpose of this report is to present the results of a parametric study on how the RMS errors in position and velocity are affected by varying the uncertainties in range rate and angles.

I. ORBITAL PARAMETERS

The orbit used for this study is a typical Apollo Earth Parking Orbit with a launch azimuth of 90° . It is nearly circular and has an orbital height of approximately 200 km (110 nautical miles). The following are the orbital parameters:

Epoch: 8/5/61 $0^{\text{h}}5^{\text{m}}0^{\text{s}}.6$

$a = 6625.6 \text{ km}$	$X = -2725.6266$	km
$e = 0.00712$	$Y = -5112.7247$	
$i = 28^\circ 5'$	$Z = 3115.5562$	
$\Omega = 143^\circ 984$	$\dot{X} = 6.7634412$	
$\omega = 97^\circ 0'$	$\dot{Y} = -3.8824499$	km/sec
$M = 0^\circ 0'$	$\dot{Z} = -0.4542630$	

II. TRACKING SYSTEM

Range rate and angles are measured every second by a tracking ship located at $26^\circ 40'$ North Latitude and $49^\circ 20'$ West Longitude. This location was used in order to have good tracking for the period when the spacecraft is inserted into the parking orbit. Uncertainties in range rate of $\pm 0.1 \text{ m/s}$ and $\pm 0.03 \text{ m/s}$ are assumed and combined in turn with each of three sets of angular uncertainties: $\pm 2 \times 10^{-3} \text{ radians}$, $\pm 6 \times 10^{-4} \text{ radians}$, and $\pm 2 \times 10^{-4} \text{ radians}$. Additional solutions incorporate a total tracker location uncertainty of approximately $\pm 450 \text{ meters}$.

III. TRACKING MODE

Since there are variations in the length of time a radar searches before it acquires a spacecraft, this searching or acquisition mode has been simulated for the present study (see Figure 1). The actual amount of tracking time varies from the maximum (tracking during the entire interval that the spacecraft is possibly visible to the tracker), which implies immediate acquisition, to no tracking at all. Between these situations are the cases where the maximum possible tracking time is shortened by the amount of acquisition time.

Therefore, solutions were made for various acquisition times, starting with acquisition as soon as the spacecraft is first possibly visible to the tracker, and advancing the acquisition time in 30-second steps. In each case, tracking is continued to the time of last possible visibility and errors in position and velocity of the spacecraft are evaluated at that time.

The total pass over the tracker is approximately 5-1/4 minutes.

IV. SUMMARY OF RESULTS

The RMS errors in position and velocity of the spacecraft are summarized in Tables 1 and 2, respectively. Figures 2-33 compare graphically how the RMS errors in position and velocity are affected by: 1) changing the uncertainty in angles; 2) changing the uncertainty in range rate; and 3) including the uncertainty in station location.

Position RMS Errors

With optimum tracking (i.e., immediate acquisition of the spacecraft and tracking over the full 5-1/4 minutes of the pass), when $\delta \dot{r} = \pm 0.1$ m/s and $\delta\alpha = \delta\epsilon = \pm 2$ milliradians, the RMS error in position is 144.2 meters. When the angular uncertainty is decreased to $\pm .6$ milliradians, the position error is 44.1 meters; to $\pm .2$ milliradians 16.6 meters. Additional figures are given in Table 1 for the position errors when 1, 2, 3 or 4 minutes are needed for the radar to acquire the spacecraft. It will be noted that if the angular uncertainty is improved by a factor of 3 or 10, the position error is improved by approximately the same factor.

The RMS errors in position described above are illustrated by Figure 2, which contains three curves, one for each of the angular uncertainties, showing clearly how the angles affect the RMS errors in position.

TABLE 1
RMS ERRORS IN POSITION OF SPACECRAFT
APOLLO PARKING ORBIT (A90 H200)
(IN METERS)

ACTUAL TRACKING TIME (MIN.)	$\delta_a = \delta\epsilon = \pm 2 \times 10^{-3}$ radians			$\delta_a = \delta\epsilon = \pm 6 \times 10^{-4}$ radians			$\delta_a = \delta\epsilon = \pm 2 \times 10^{-4}$ radians		
	$\delta_i = \pm 0.1$ m/s	$\delta_i = \pm 0.03$ m/s	$\delta_i = \pm 0.1$ m/s	$\delta_i = \pm 0.03$ m/s	$\delta_i = \pm 0.1$ m/s	$\delta_i = \pm 0.03$ m/s	$\delta_i = \pm 0.1$ m/s	$\delta_i = \pm 0.03$ m/s	$\delta_i = \pm 0.1$ m/s
A. Not including station location errors									
1.25	1102.1	656.9	484.4	330.6	327.5	153.8			
2.25	262.8	237.0	101.1	78.9	38.3	33.1			
3.25	188.7	177.2	70.5	56.6	26.5	23.0			
4.25	156.8	156.0	49.2	47.0	19.6	16.3			
5.25	144.2	143.8	44.1	43.3	16.6	14.7			
B. Including station location errors of ± 452.5 meters total									
1.25	1	1	723.8	612.6	580.5	486.5			
2.25	536.0	518.2	460.2	454.5	447.4	447.1			
3.25	492.3	486.2	452.1	449.7	446.0	445.9			
4.25	477.0	476.7	448.3	448.0	445.5	445.3			
5.25	472.0	471.9	447.5	447.4	445.2	445.2			
1] No solution									

TABLE 2
RMS ERRORS IN VELOCITY OF SPACECRAFT
APOLLO PARKING ORBIT (A90 H200)
(IN METERS PER SECOND)

ACTUAL TRACKING TIME (MIN.)	$\delta\alpha = \delta\epsilon = \pm 2 \times 10^{-3}$ radians			$\delta\alpha = \delta\epsilon = \pm 6 \times 10^{-4}$ radians			$\delta\alpha = \delta\epsilon = \pm 2 \times 10^{-4}$ radians		
	$\delta\dot{r} = \pm 0.1$ m/s	$\delta\dot{r} = \pm 0.03$ m/s	$\delta\dot{r} = \pm 0.1$ m/s	$\delta\dot{r} = \pm 0.03$ m/s	$\delta\dot{r} = \pm 0.1$ m/s	$\delta\dot{r} = \pm 0.03$ m/s	$\delta\dot{r} = \pm 0.1$ m/s	$\delta\dot{r} = \pm 0.03$ m/s	$\delta\dot{r} = \pm 0.1$ m/s
A. Not including station location errors									
1.25	9.10	5.39		3.09		2.73		1.16	1.02
2.25	1.98	1.63		.860		.595		.324	.280
3.25	1.19	1.10		.463		.356		.176	.151
4.25	.920	.915		.291		.276		.119	.096
5.25	.821	.819		.252		.246		.097	.084
B. Including station location errors of ± 452.5 meters total									
1.25		<u>1</u>		3.21		2.94		1.28	1.14
2.25		<u>2.08</u>		.968		.746		.513	.498
3.25		1.27		1.17		.612		.430	.423
4.25		1.00		.996		.488		.409	.404
5.25		.910		.908		.466		.404	.401
[1] No solution									

Table 1 also gives the RMS errors in position when $\delta \dot{r} = \pm 0.03$ m/s and the angular uncertainties and acquisition times are as given above. Here again it is noted that the position error improves proportionately to the improvement in angular uncertainty. The accompanying illustration is Figure 3, which is similar to Figure 2.

Next we consider how the uncertainty in range rate affects the RMS error in position. The data given in Table 1 are also presented in Figures 4, 5, and 6 (one for each of the angular uncertainties), comparing this time by the uncertainty in range rate. Curve A on each figure is for $\delta \dot{r} = \pm 0.1$ m/s and curve B for $\delta \dot{r} = \pm 0.03$ m/s. These figures demonstrate very clearly that if the space-craft can be acquired within two, or even three, minutes and tracked for two or three minutes, the position error is practically the same whether $\delta \dot{r} = \pm 0.1$ m/s or ± 0.03 m/s.

Section B of Table 1 gives the RMS errors in position when station location uncertainties totaling 450 meters are included with each of the sets of uncertainties explained above. It will be noted that the angular uncertainty does not play as important a role as in the cases where the station location uncertainty was not considered. When $\delta \dot{r} = \pm 0.1$ m/s, the RMS error in position, assuming optimum tracking, is 472.0 meters, 447.5 meters, and 445.2 meters for the three sets of angular uncertainties in decreasing order. In other words, the position error does not decrease proportionately with the decrease in angular uncertainty, as further shown in Figure 7. Similarly, when $\delta \dot{r} = \pm 0.03$ m/s, the RMS error in position is 471.9 meters, 447.4 meters, and 445.2 meters (illustrated by Figure 8). Figures 7 and 8 also demonstrate that, with even a small amount of tracking, the RMS errors approach the level of the station location error.

Figures 9, 10, and 11 are similar to Figures 4, 5, 6 and support the conclusion that no appreciable increase in accuracy is obtained by improving the range rate.

A third type of comparison which can be drawn from Table 1 is illustrated by Figures 12-17, one figure for each set of uncertainty parameters. Each figure has two curves, comparing errors with or without station location uncertainties and pointing out again that the station location uncertainty is the dominant influence.

Velocity RMS Errors

The RMS errors in velocity of the spacecraft were studied in the same manner as were the errors in position. Results were similar and are summarized in Table 2. It was found that when $\delta \dot{r} = \pm 0.1$ m/s or ± 0.03 m/s, the RMS

error in velocity is .82 m/s, .25 m/s and .10 m/s for the three angular uncertainties. When station location errors are considered, these velocity errors become .91 m/s, .47 m/s, and .40 m/s, respectively. All figures here assume optimum tracking and increase only slightly for shorter periods of tracking as long as the time it takes the radar to acquire is no more than two minutes.

Figures 20-33 compare velocity errors in the same manner and order as were position errors. They are therefore self-explanatory. They also corroborate the conclusions reached above, which are restated below:

CONCLUSIONS

1. When range rate and angles are being measured, the RMS errors are more sensitive to the angles than to range rate.
2. When station location uncertainties are included, with even a small amount of tracking, the RMS errors in position approach the level of the station location uncertainty.
3. If a tracker is able to acquire a spacecraft soon enough to be able to track for approximately three minutes, the delay in acquisition does not greatly increase the RMS errors in position or velocity of the spacecraft.

ACKNOWLEDGMENT

The assistance of Thomas E. Jones in graphing all the data is gratefully acknowledged. Also Figure 1 was taken from a report by J. L. Cooley (see Reference 1 below).

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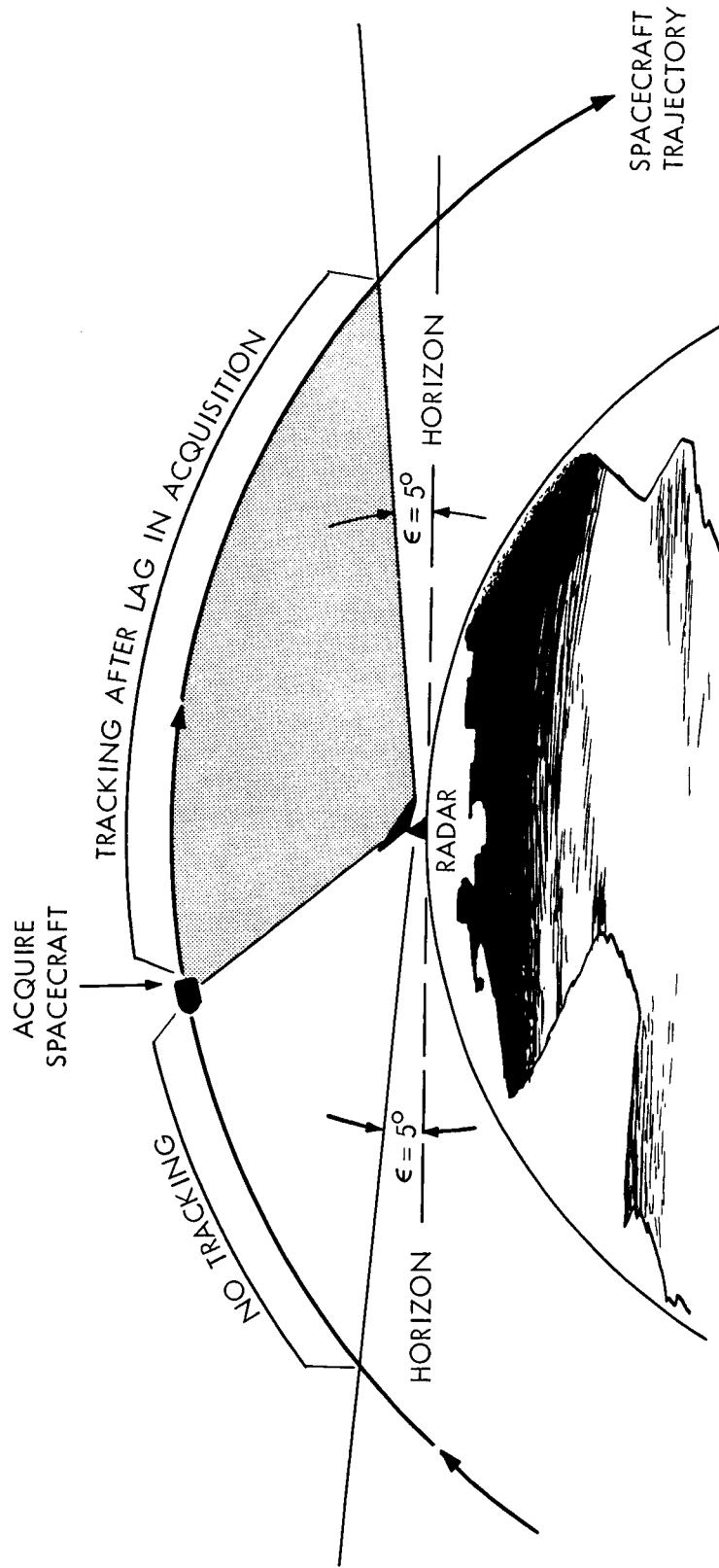


Figure 1—Acquisition Mode of Tracking

Figures 2-17

Error Propagation in Spacecraft Position Apollo Parking Orbit

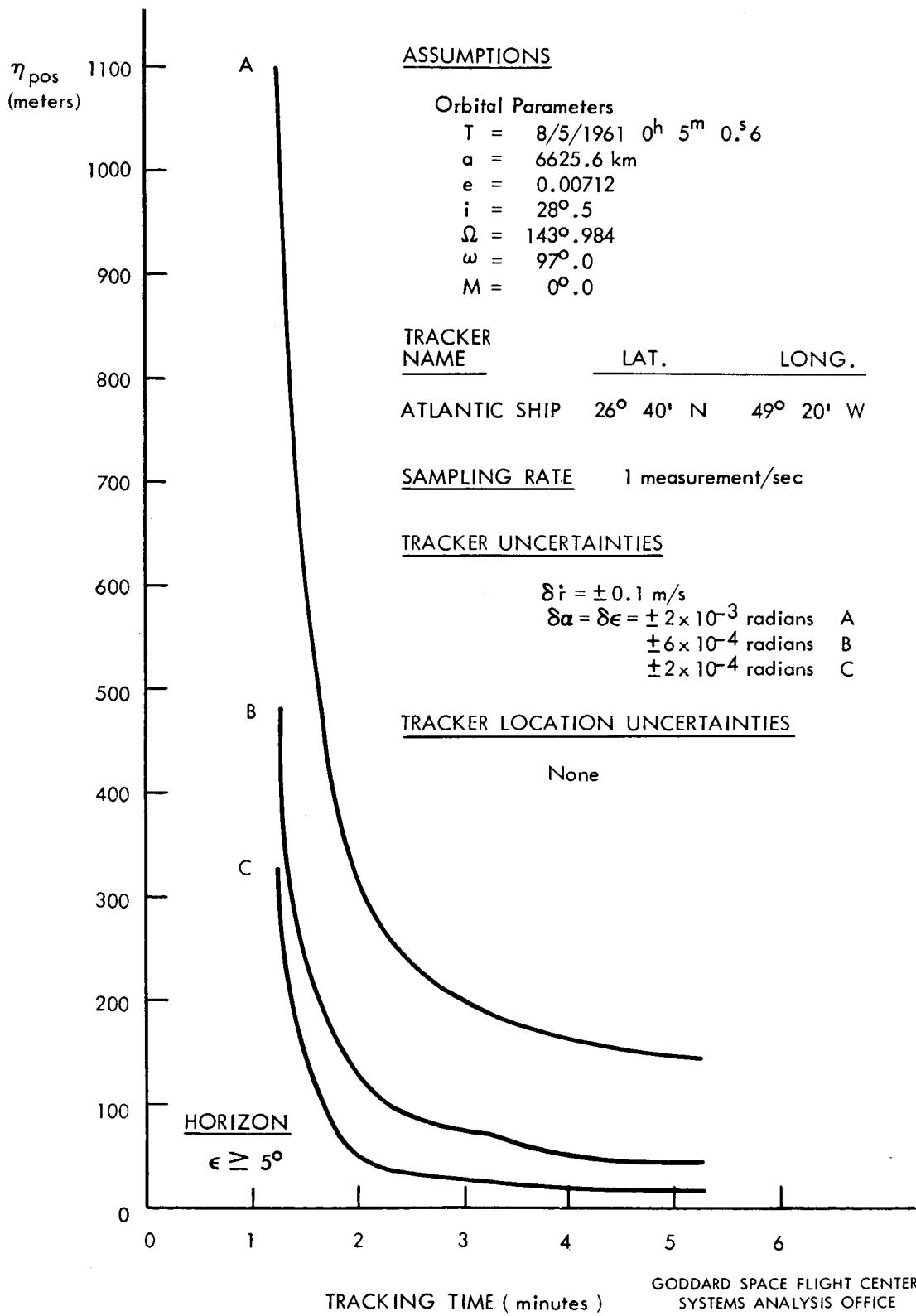


Figure 2

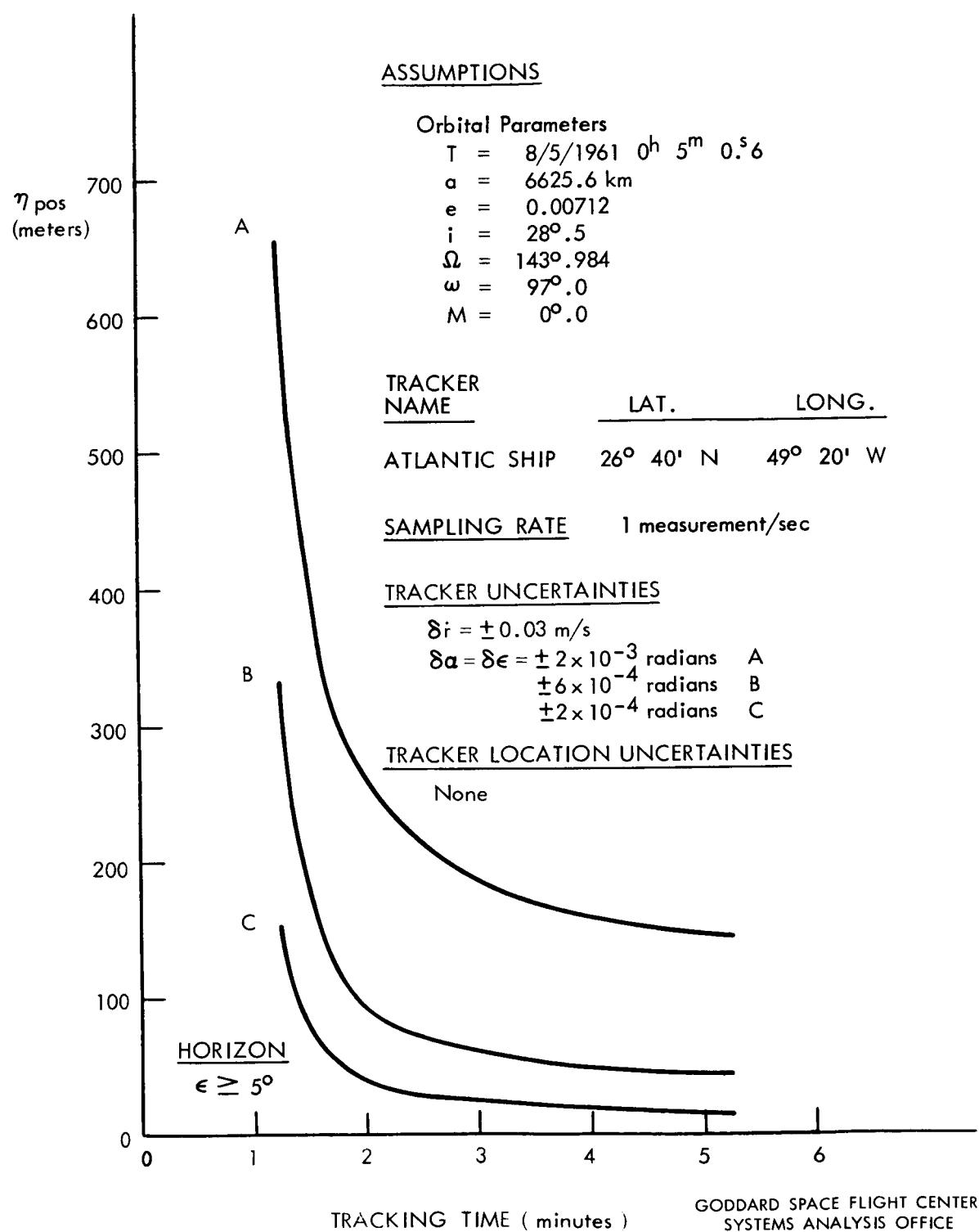


Figure 3

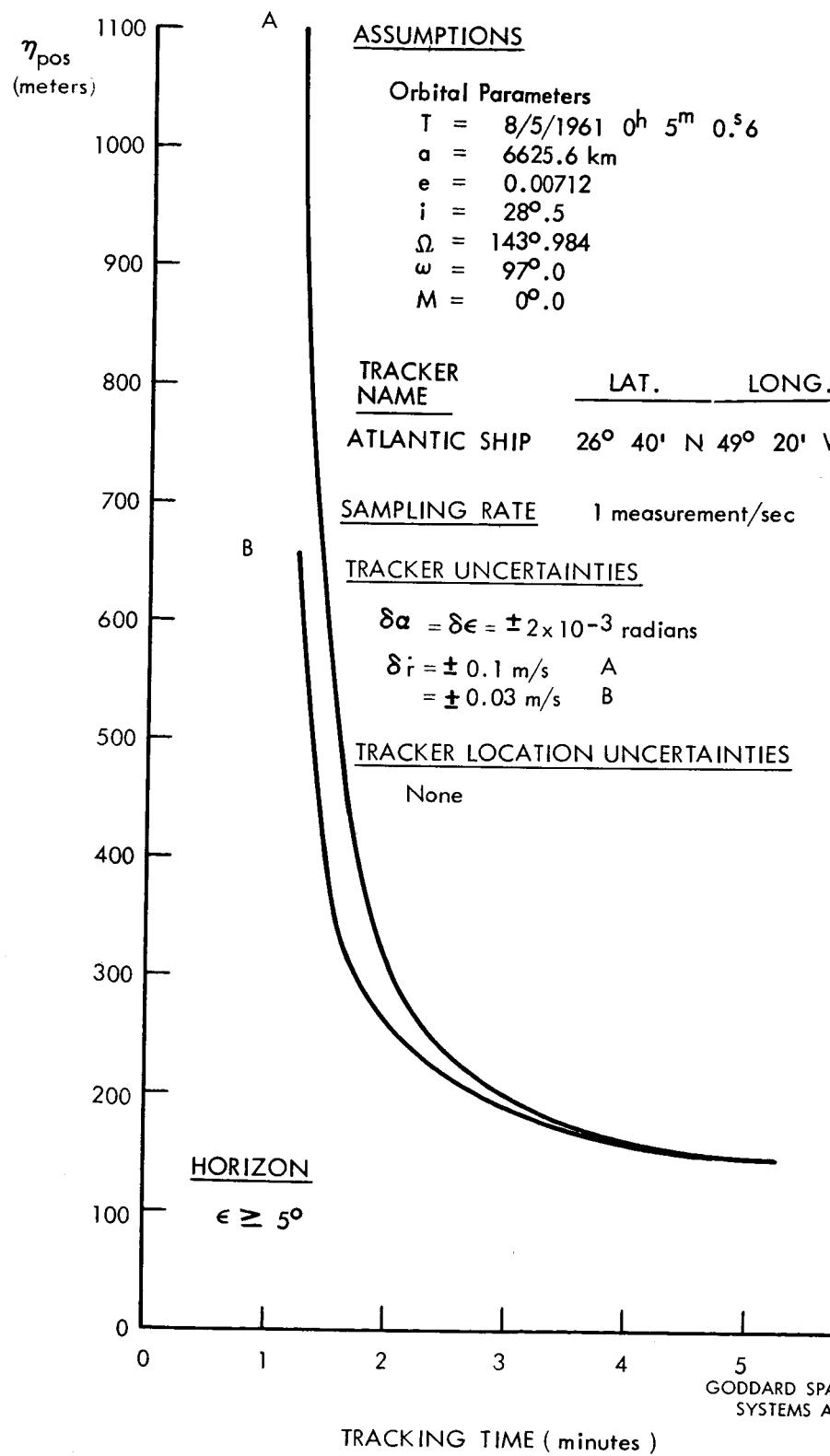


Figure 4

ASSUMPTIONS

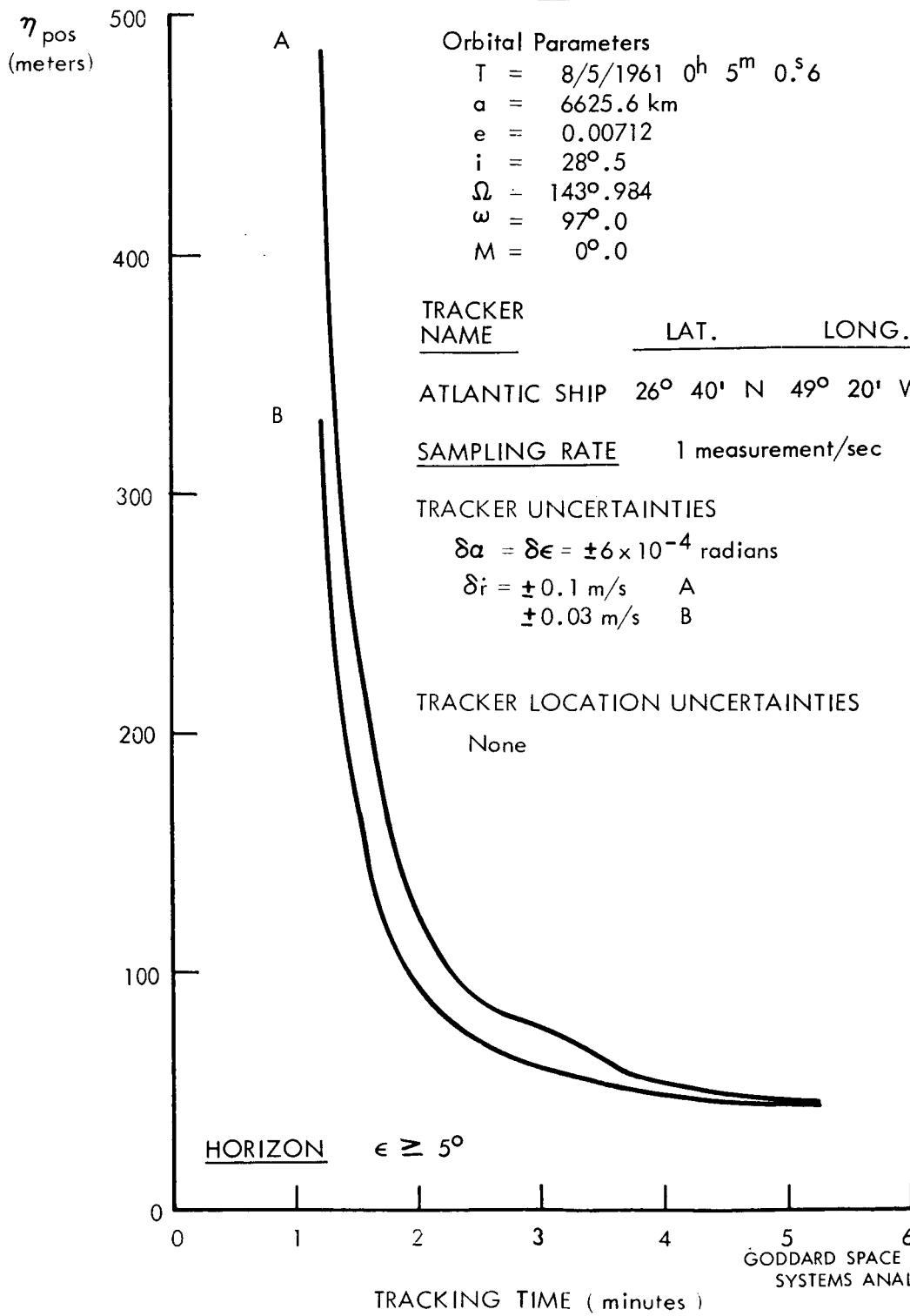


Figure 5

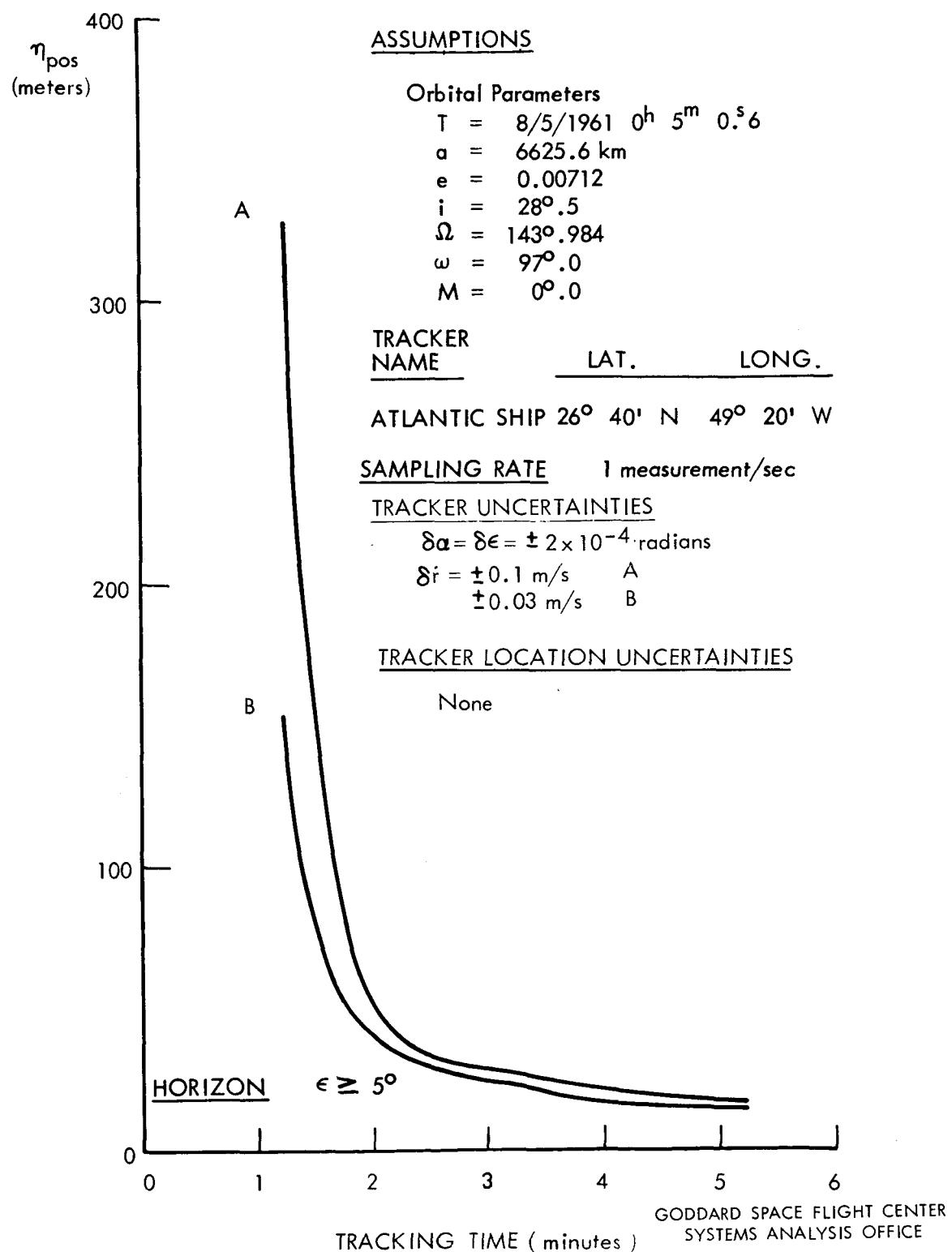


Figure 6

ASSUMPTIONS

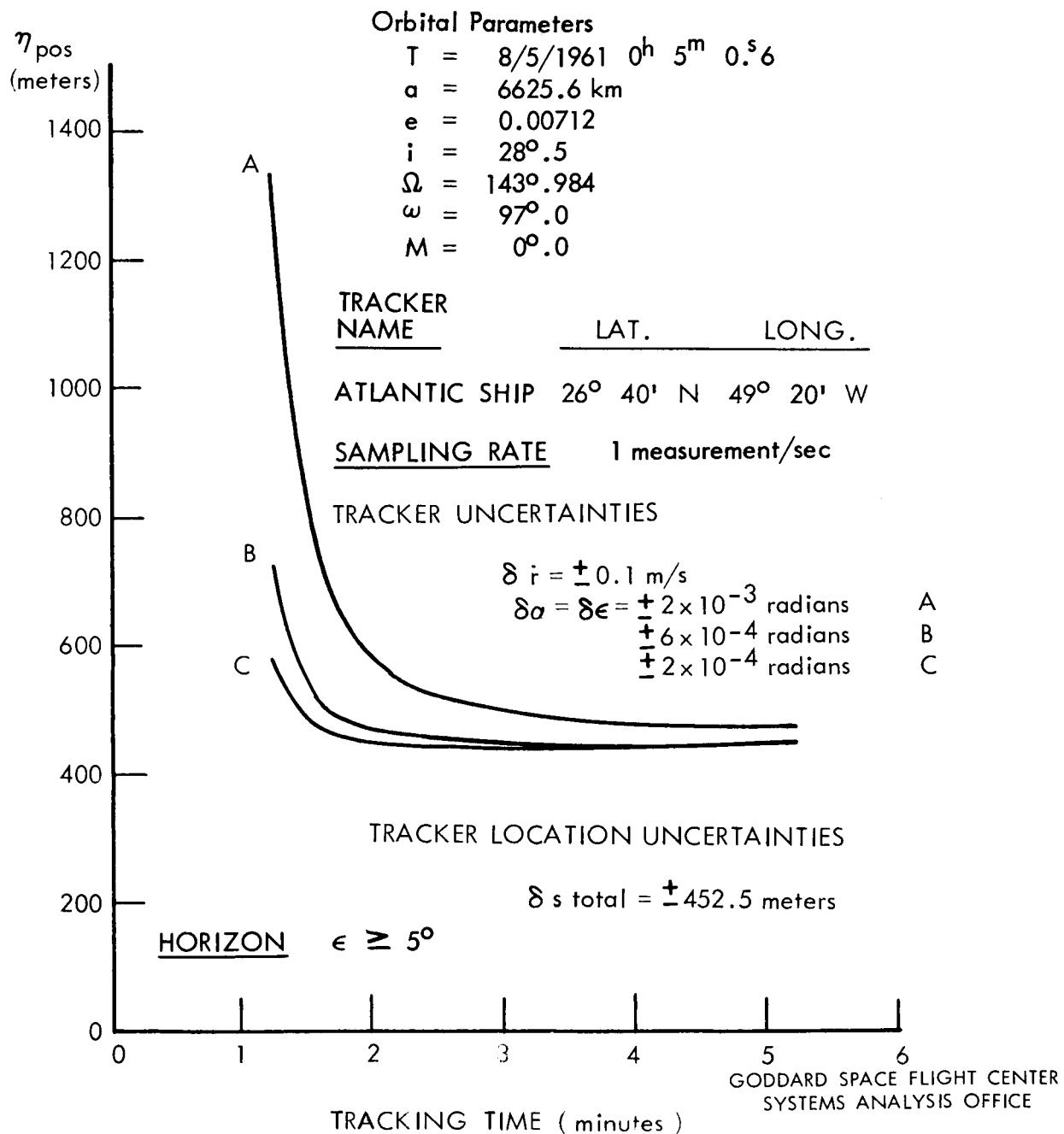


Figure 7

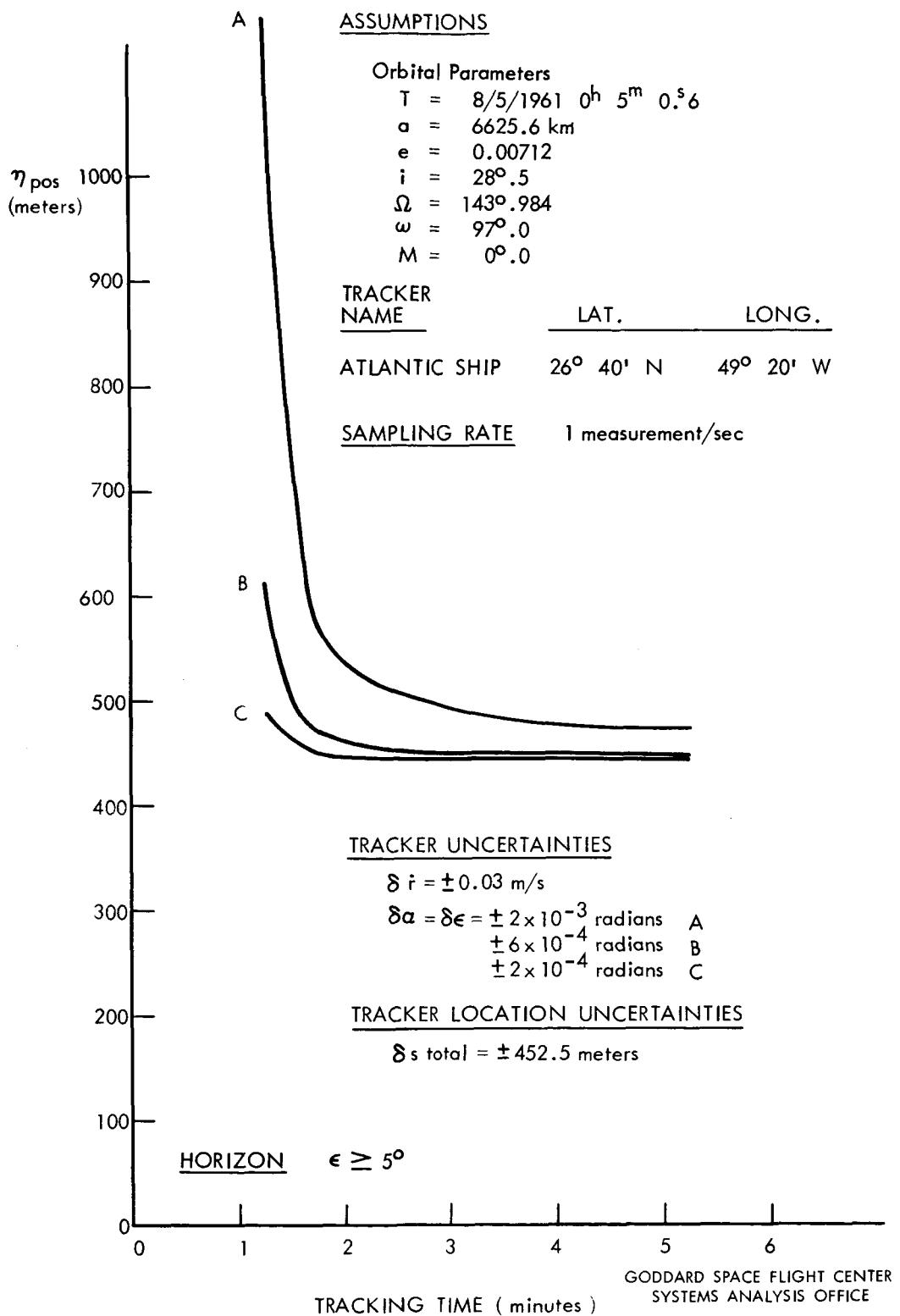


Figure 8

ASSUMPTIONS

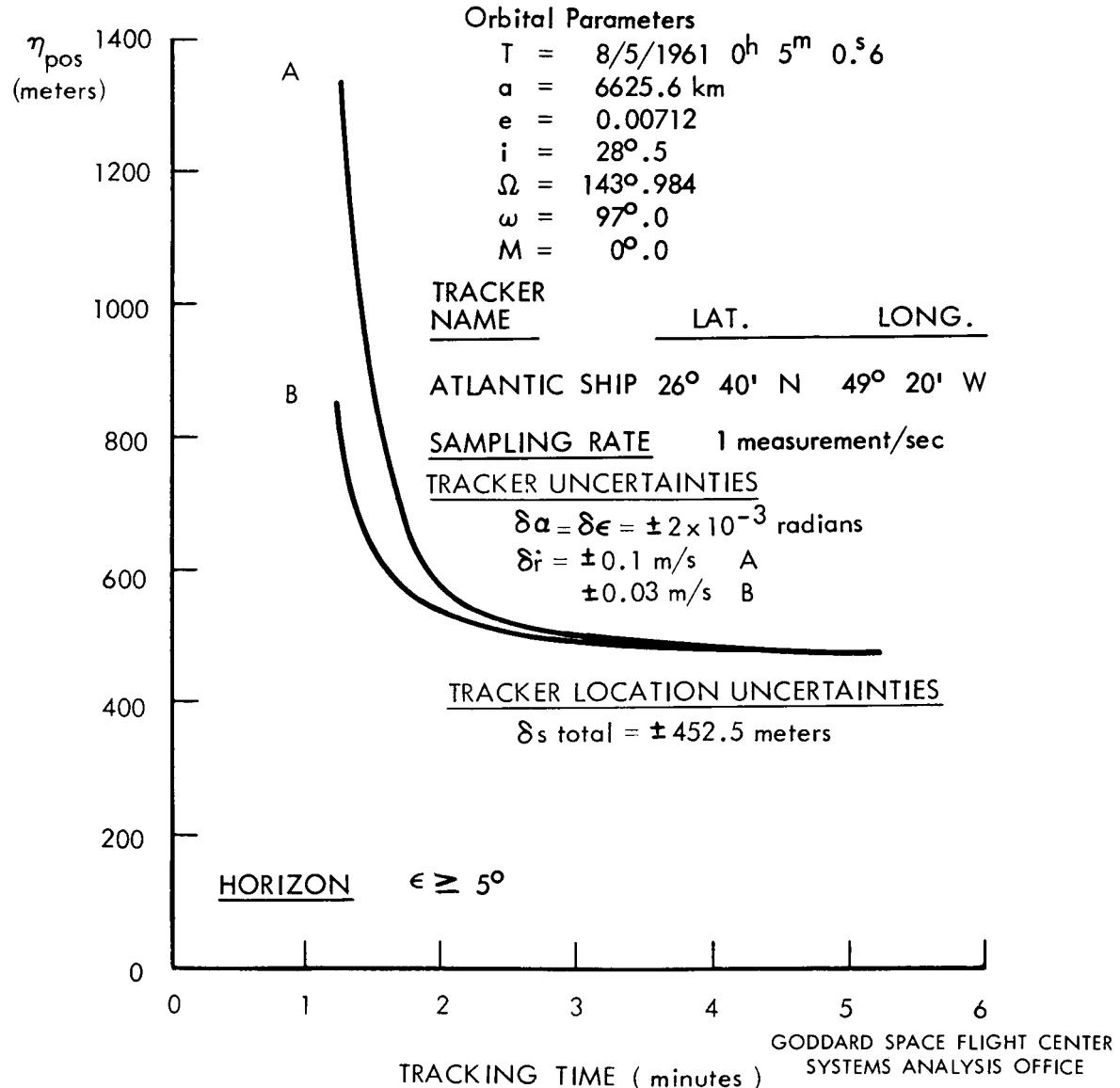


Figure 9

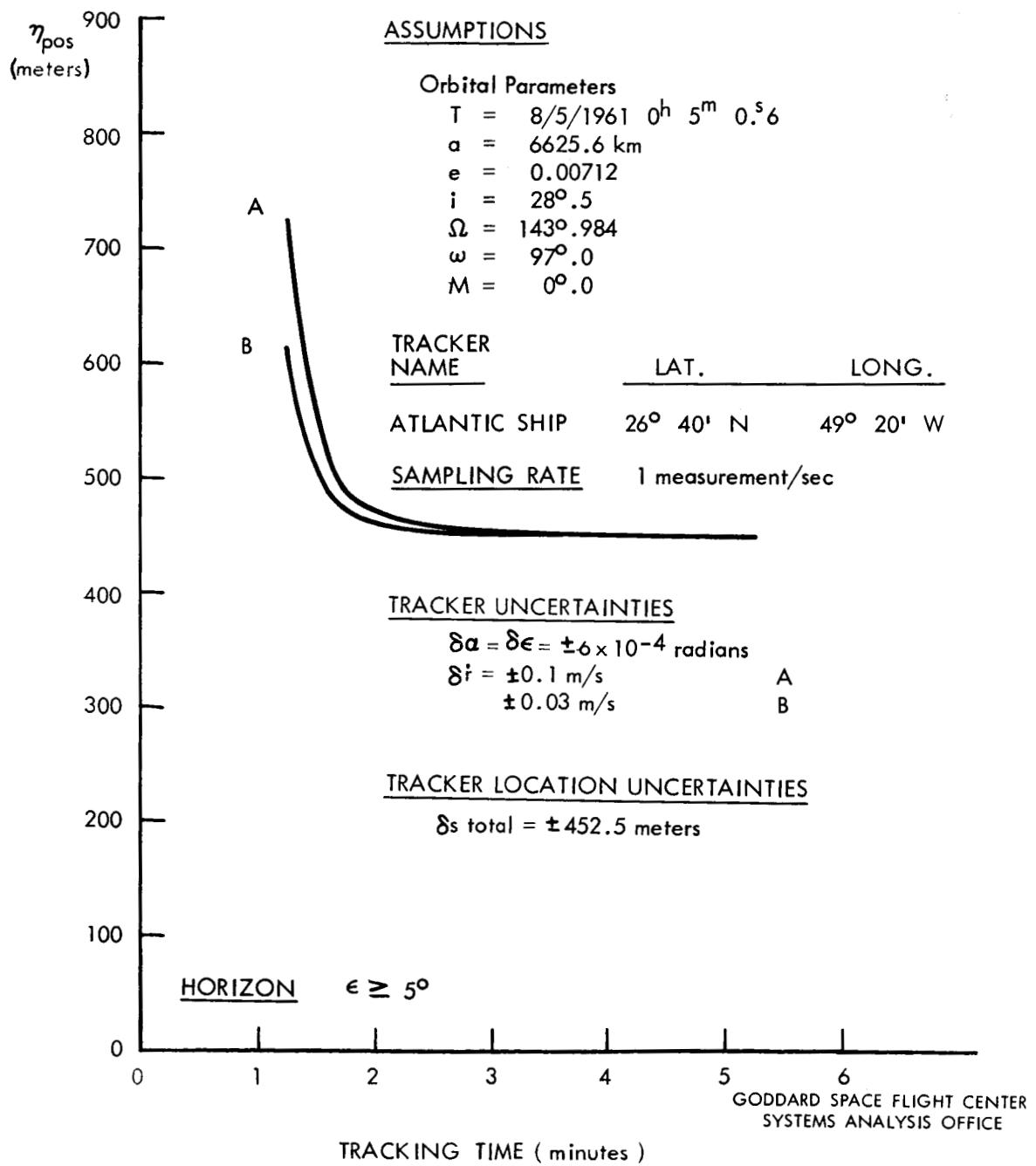


Figure 10

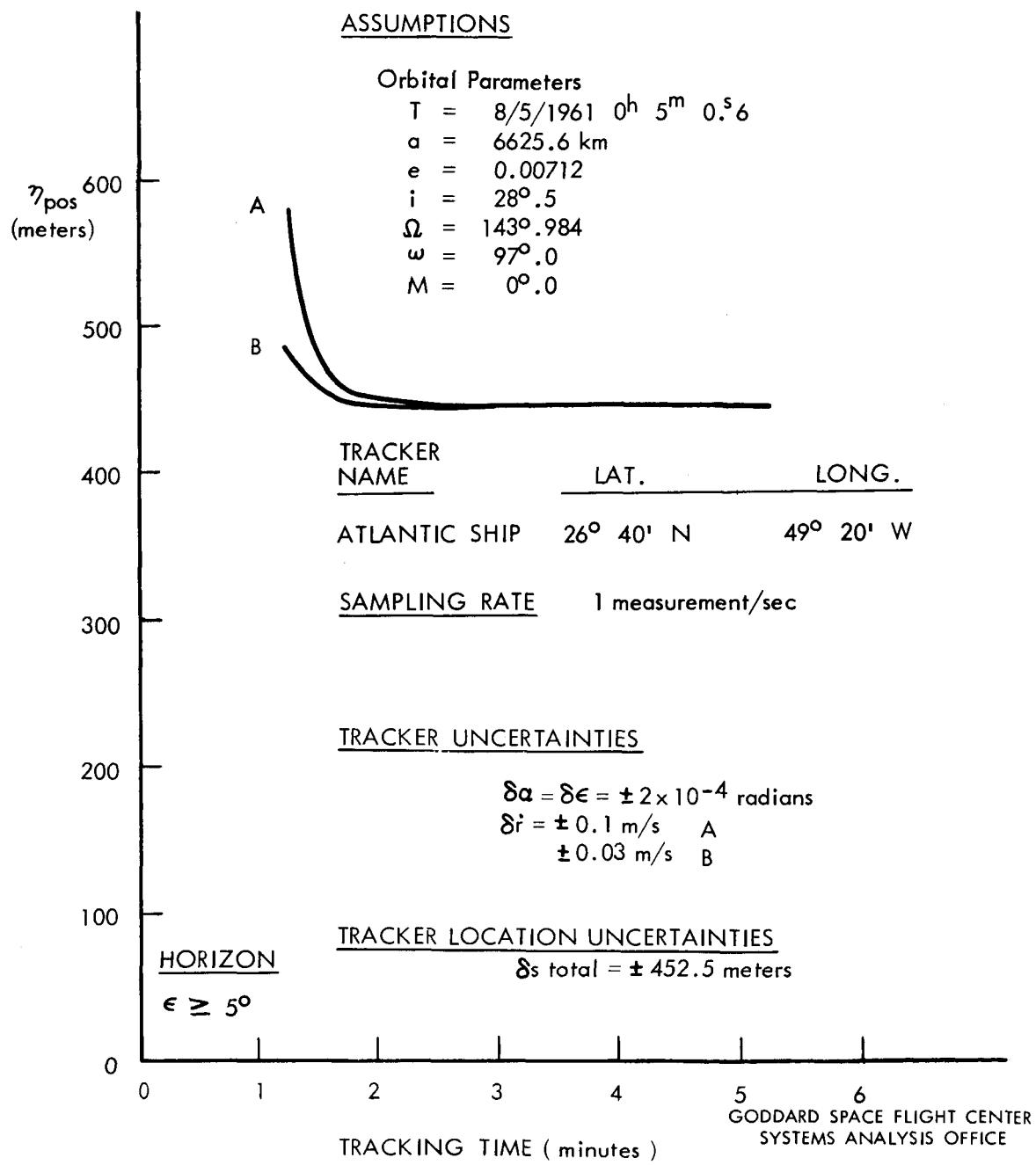


Figure 11

ASSUMPTIONS

Orbital Parameters

$T = 8/5/1961 \ 0^h \ 5^m \ 0.^s6$
 $a = 6625.6 \text{ km}$
 $e = 0.00712$
 $i = 28^\circ.5$
 $\Omega = 143^\circ.984$
 $\omega = 97^\circ.0$
 $M = 0^\circ.0$

TRACKER NAME LAT. LONG.

ATLANTIC SHIP $26^\circ 40' \text{ N}$ $49^\circ 20' \text{ W}$

SAMPLING RATE 1 measurement/sec

TRACKER UNCERTAINTIES

$$\delta i = \pm 0.1 \text{ m/s}$$

$$\delta a = \delta e = \pm 2 \times 10^{-3} \text{ radians}$$

With Station Error

TRACKER LOCATION UNCERTAINTIES

$$\delta s_{\text{total}} = \pm 452.5 \text{ meters}$$

Without Station Error

HORIZON

$$\epsilon \geq 5^\circ$$

GODDARD SPACE FLIGHT CENTER
SYSTEMS ANALYSIS OFFICE

TRACKING TIME (minutes)

Figure 12

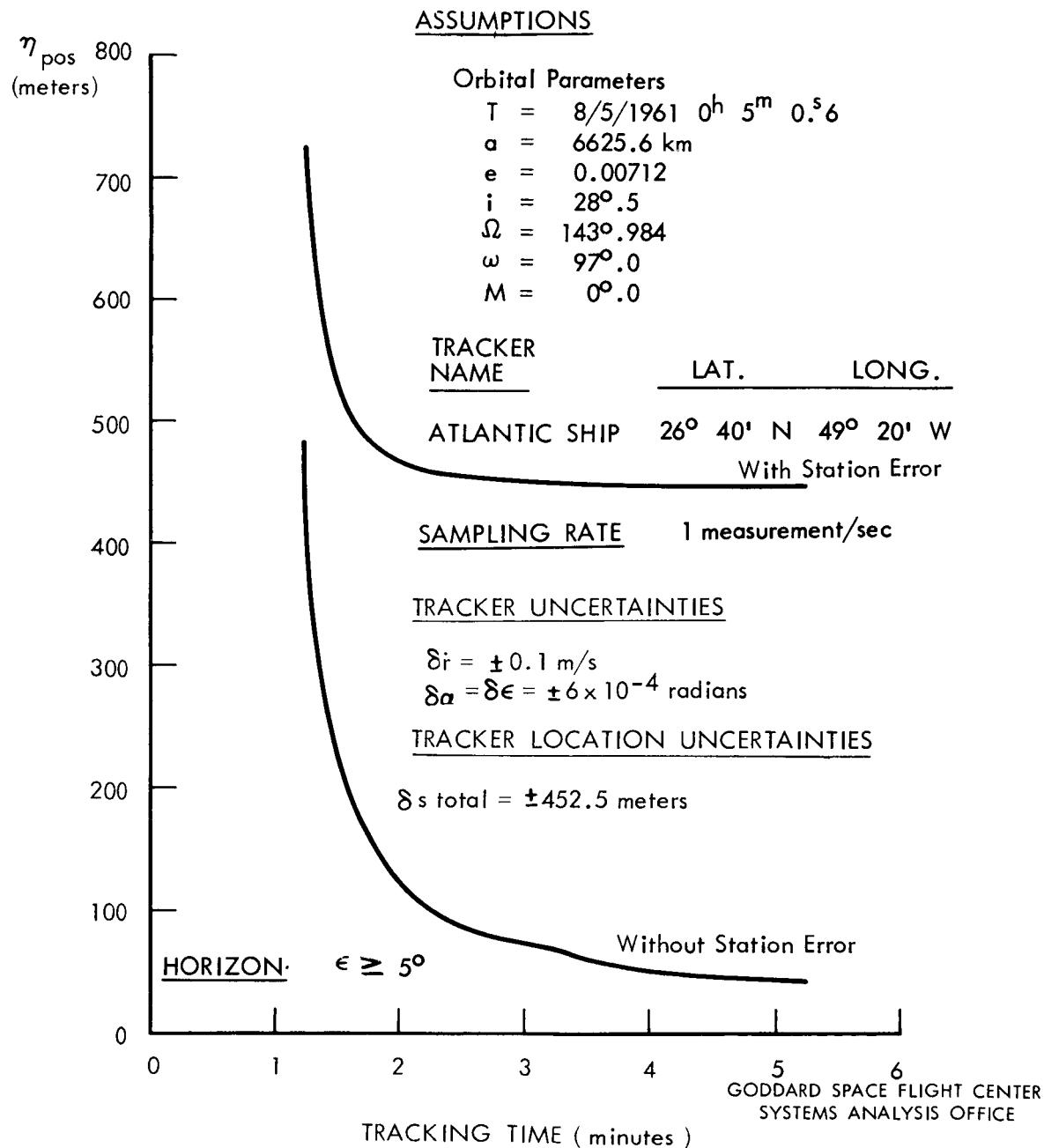


Figure 13

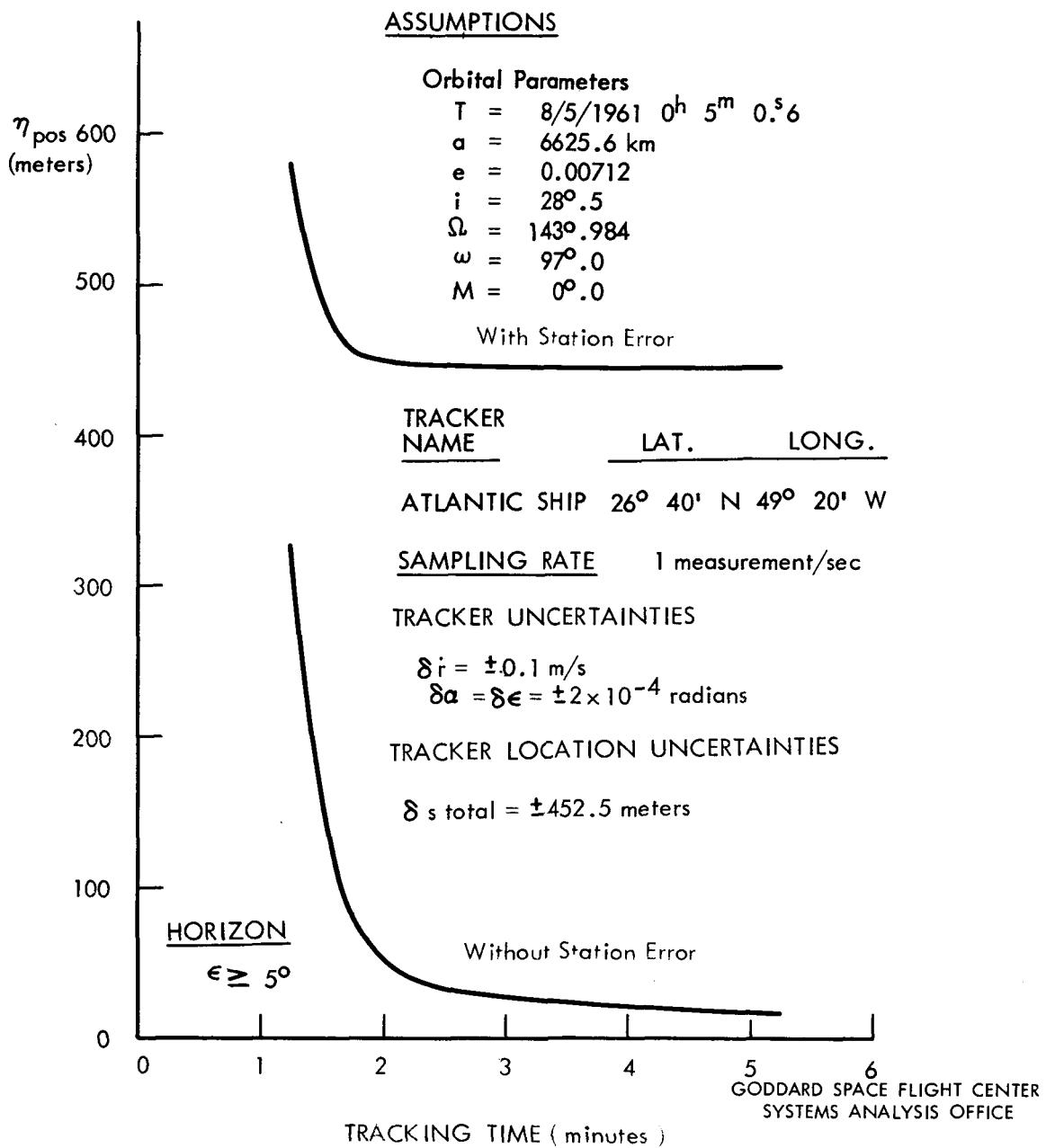


Figure 14

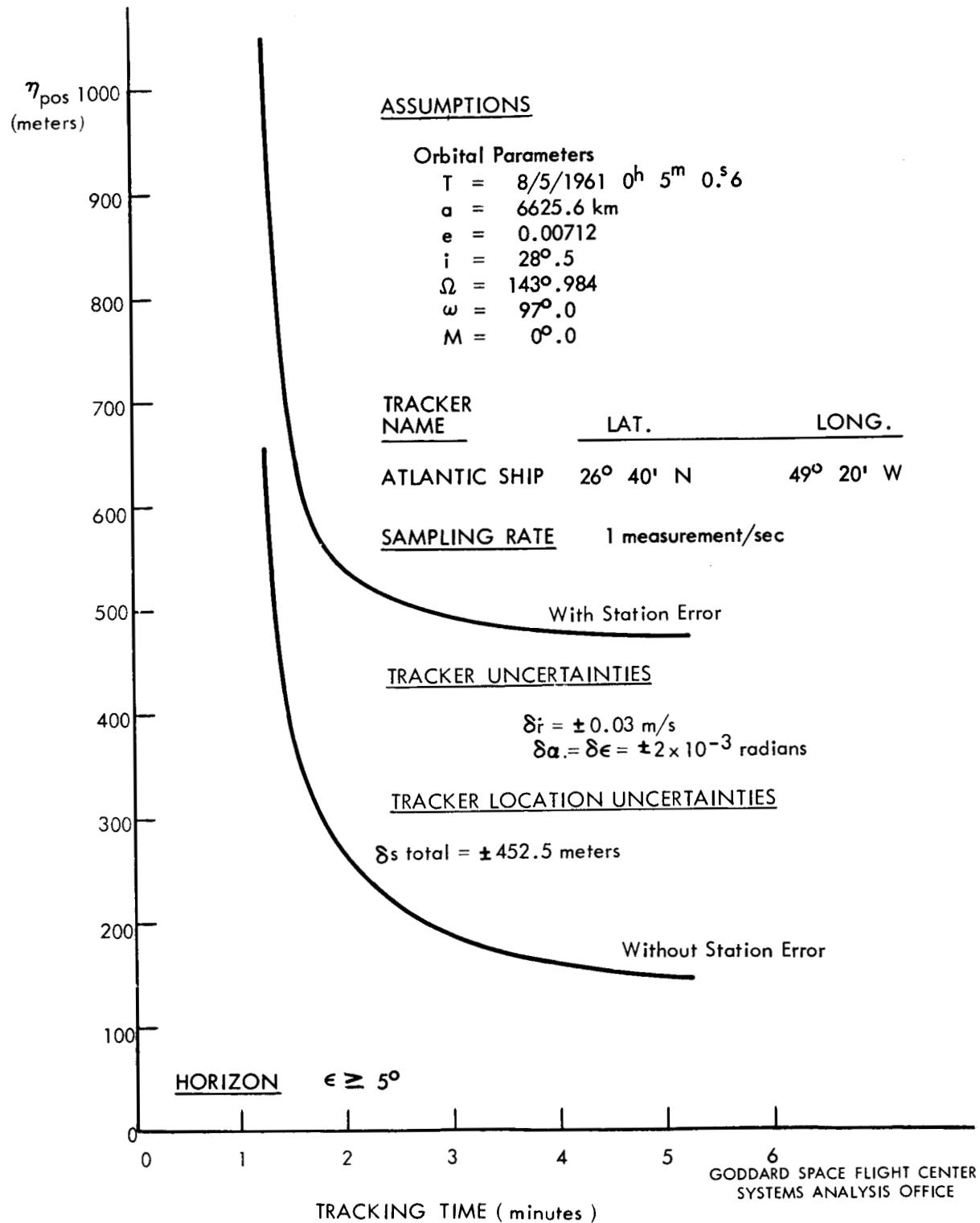


Figure 15

ASSUMPTIONS

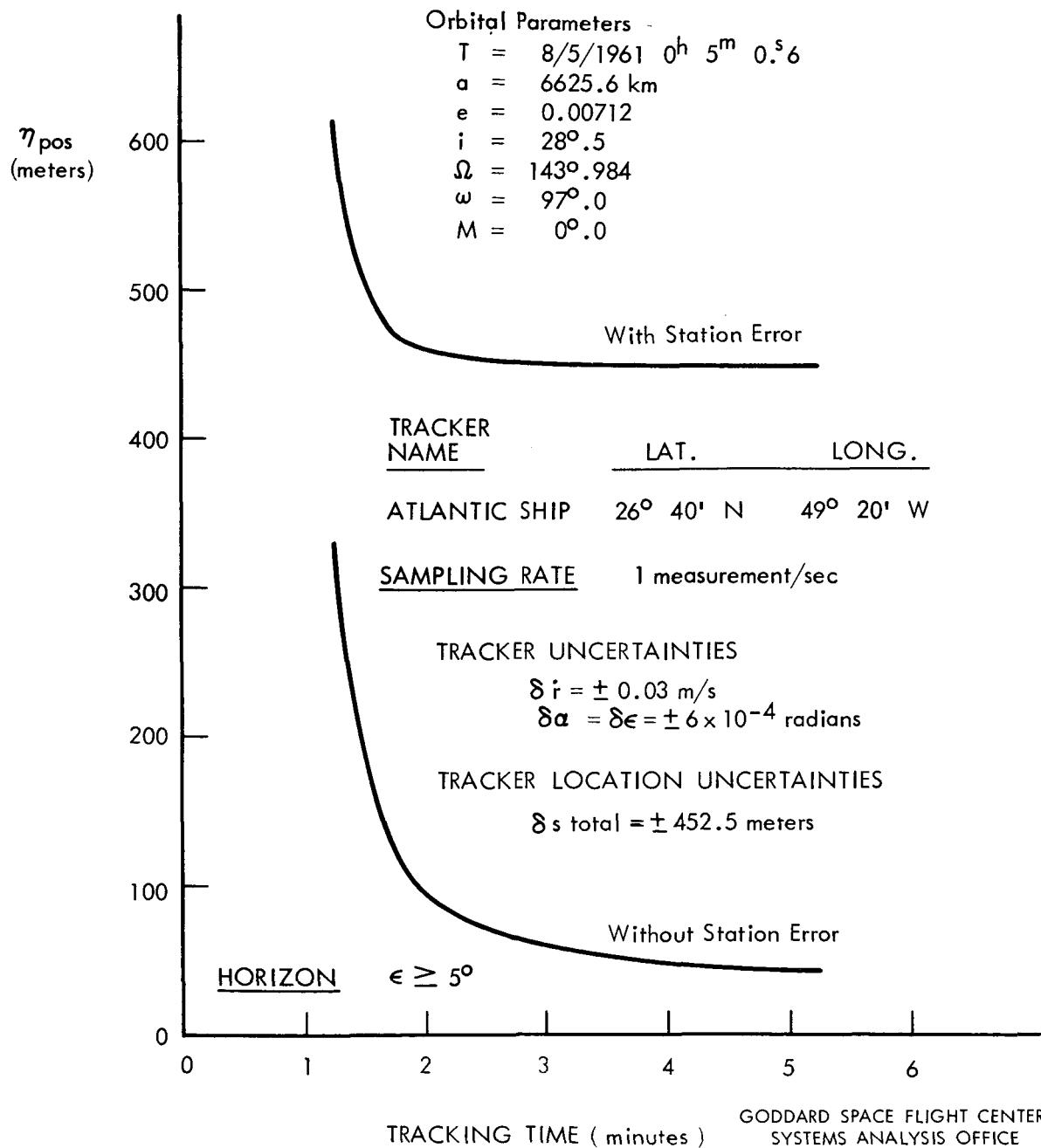


Figure 16

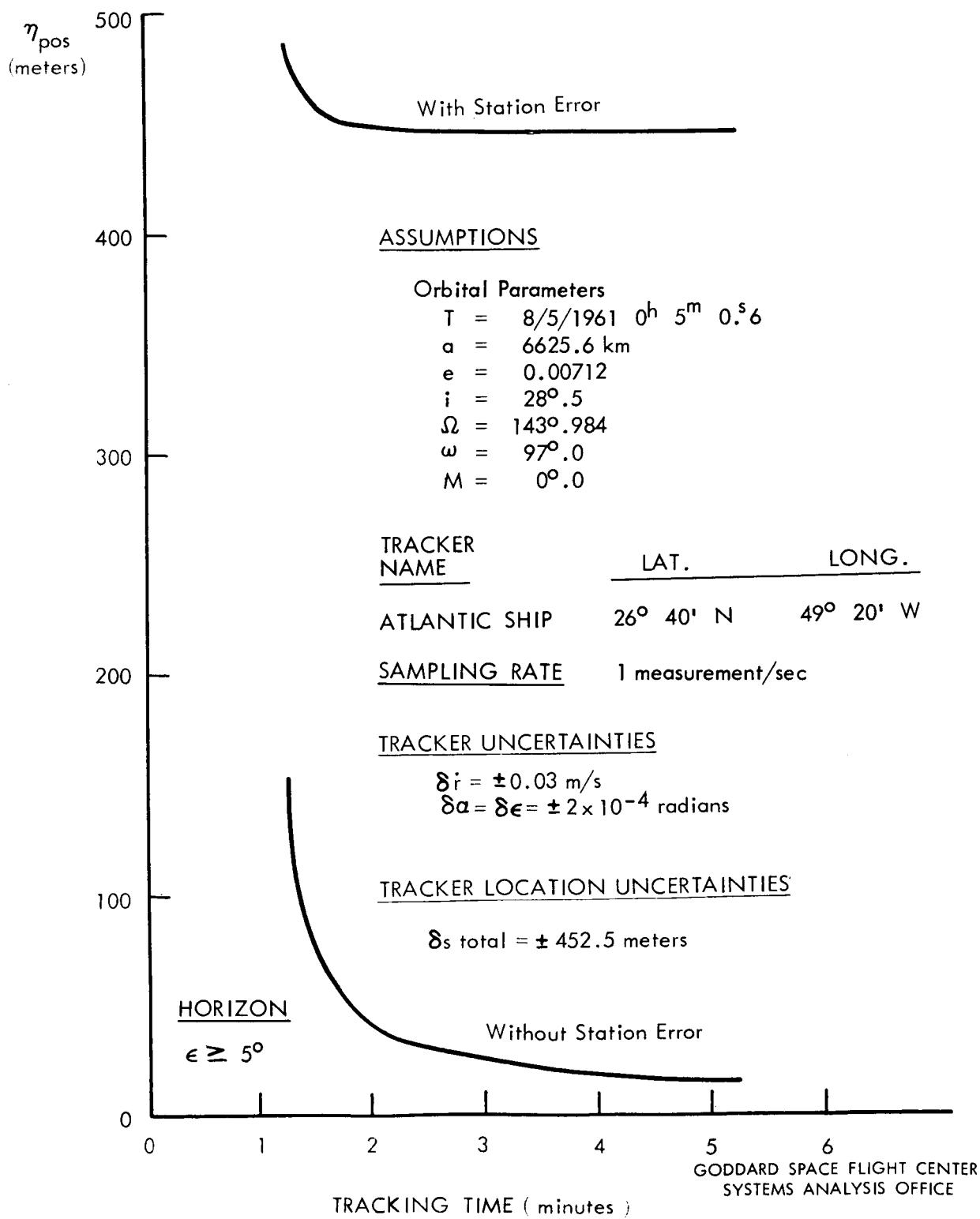


Figure 17

Figures 18-33

Error Propagation in Spacecraft Velocity Apollo Parking Orbit

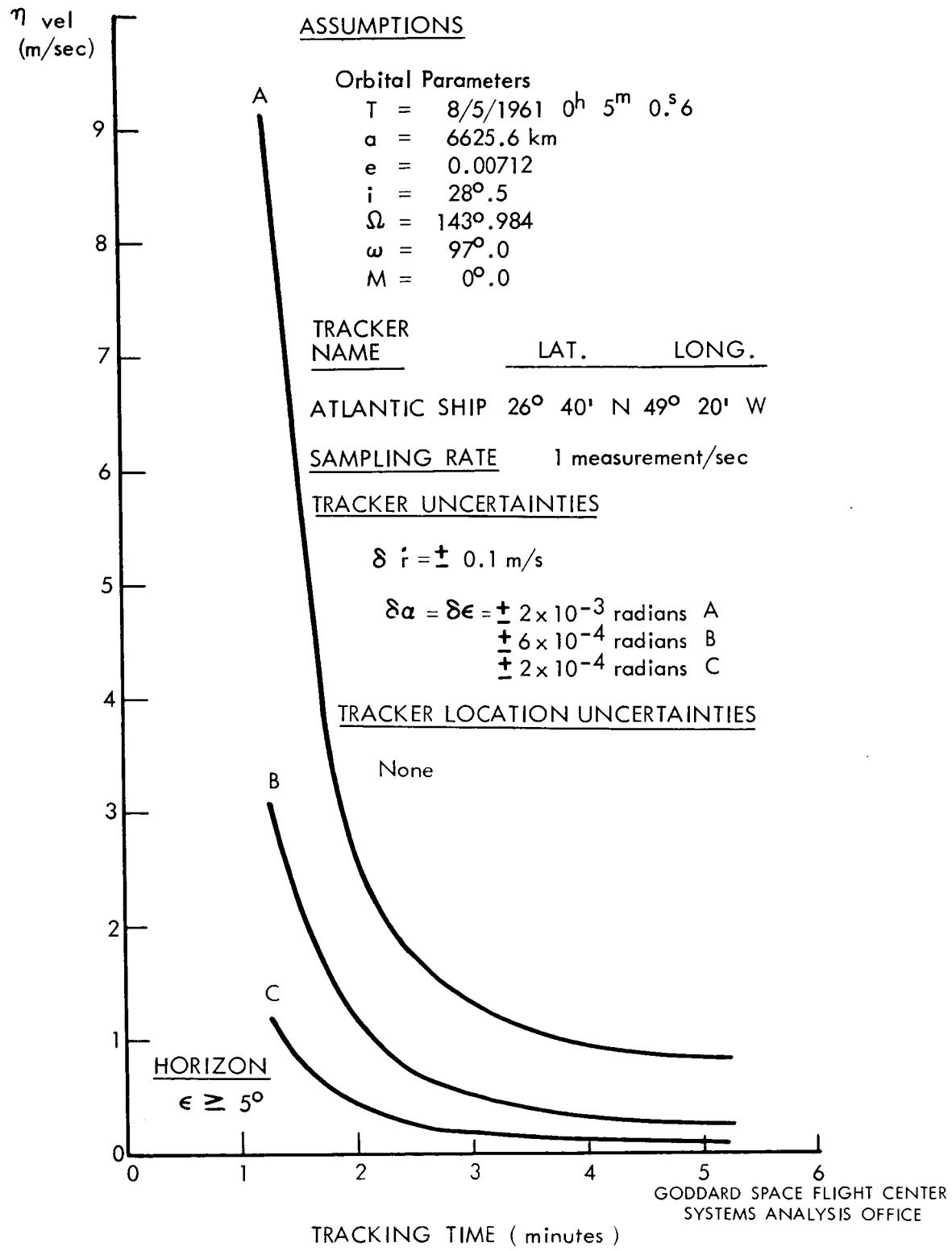


Figure 18

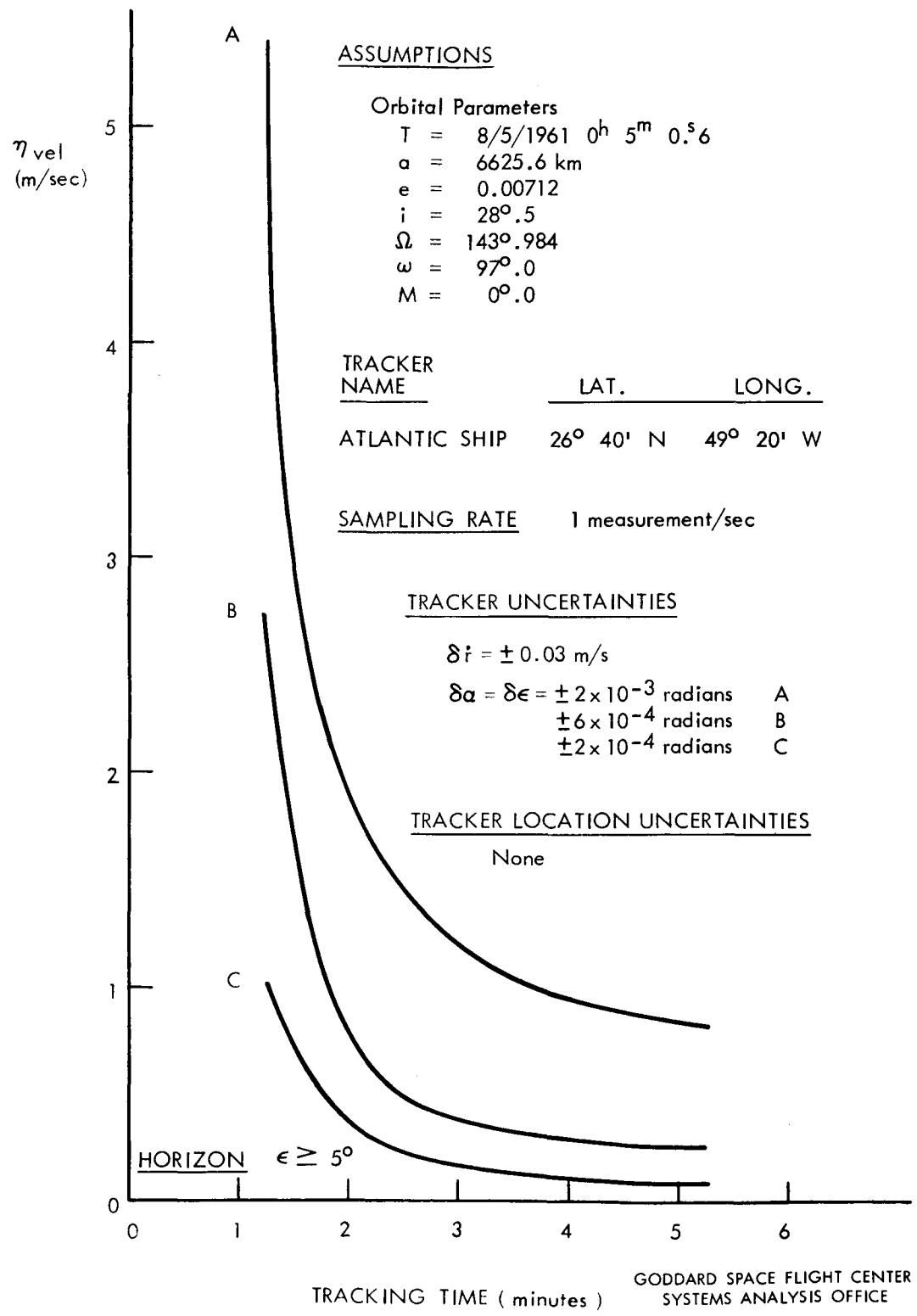


Figure 19

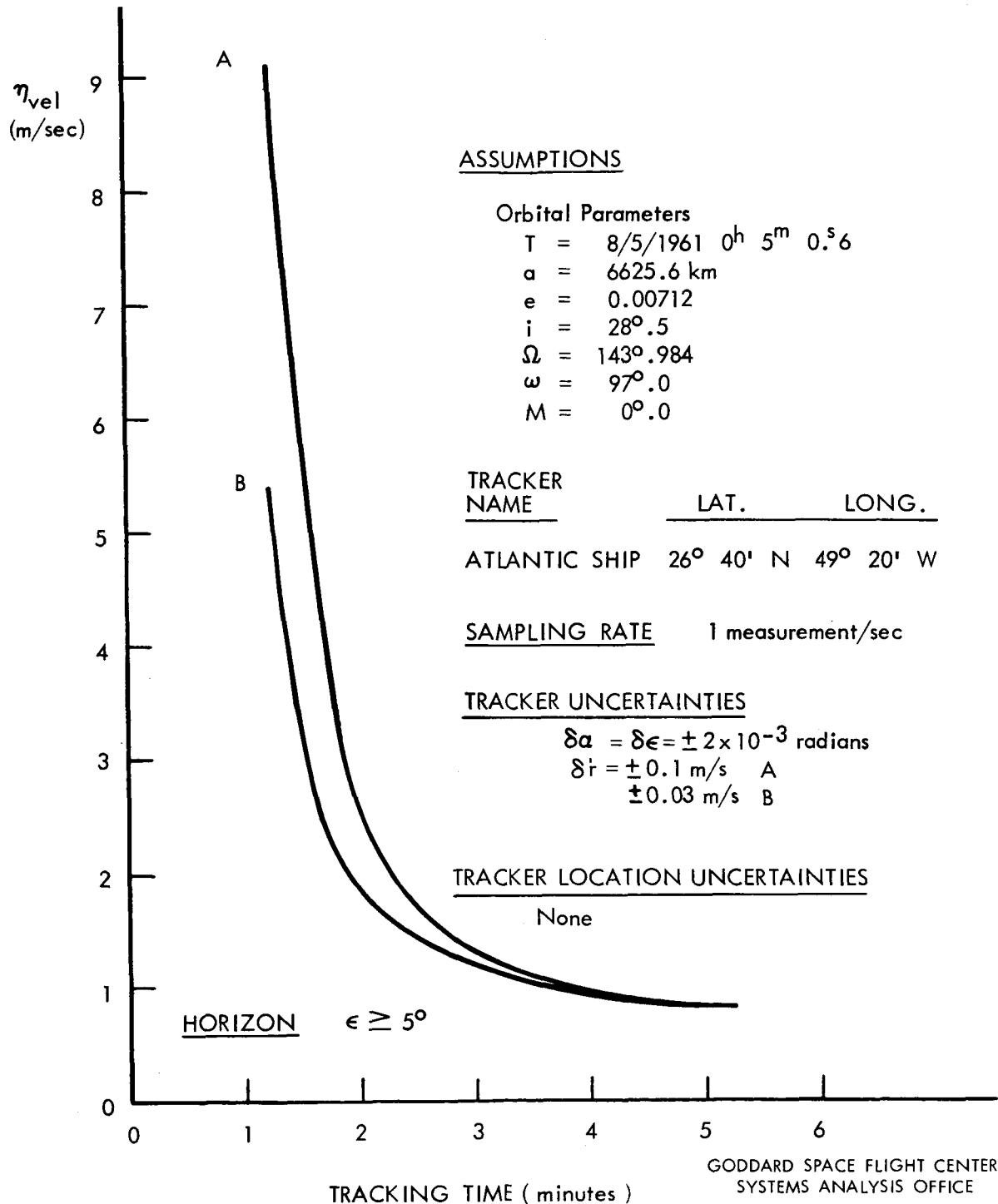


Figure 20

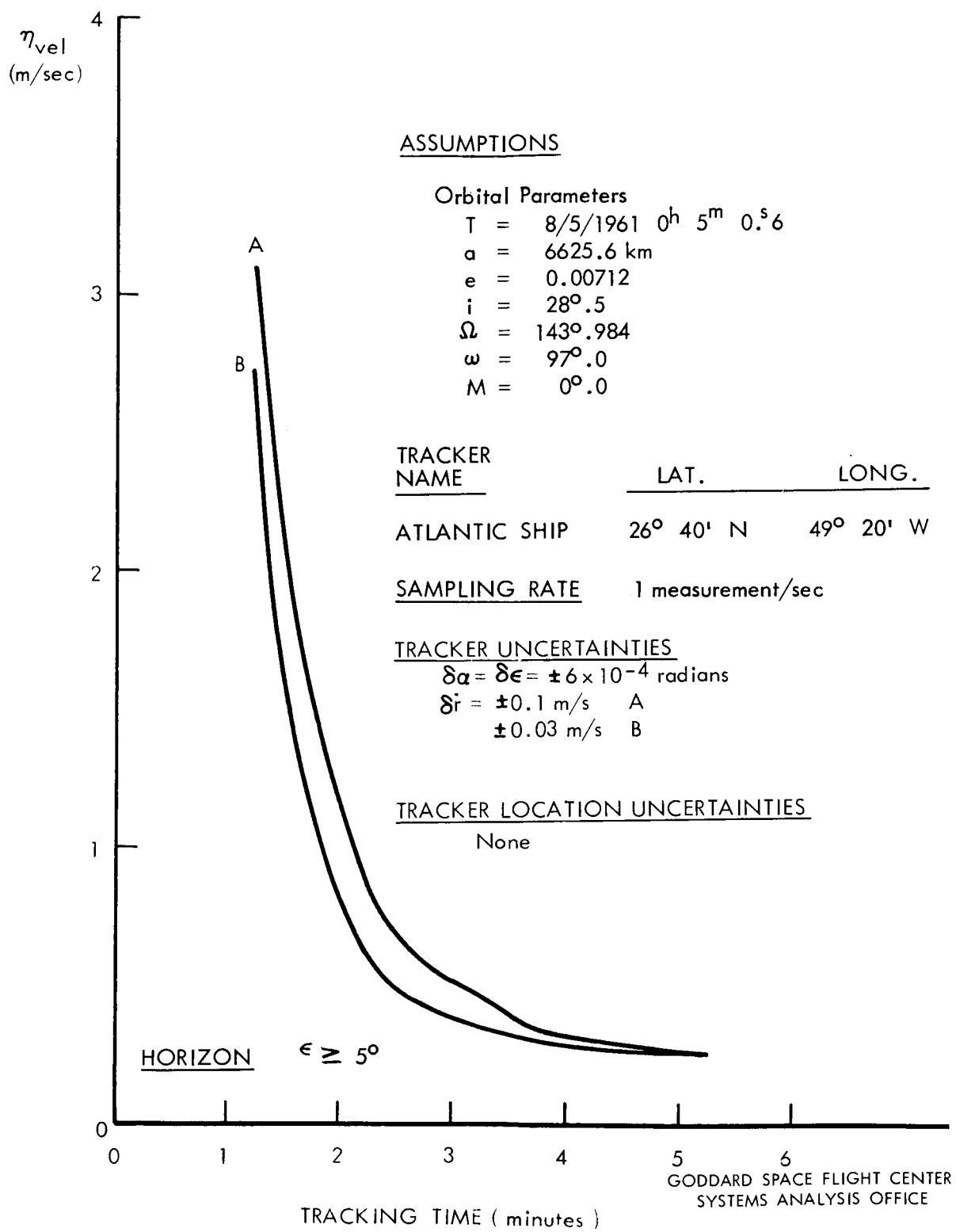


Figure 21

ASSUMPTIONS

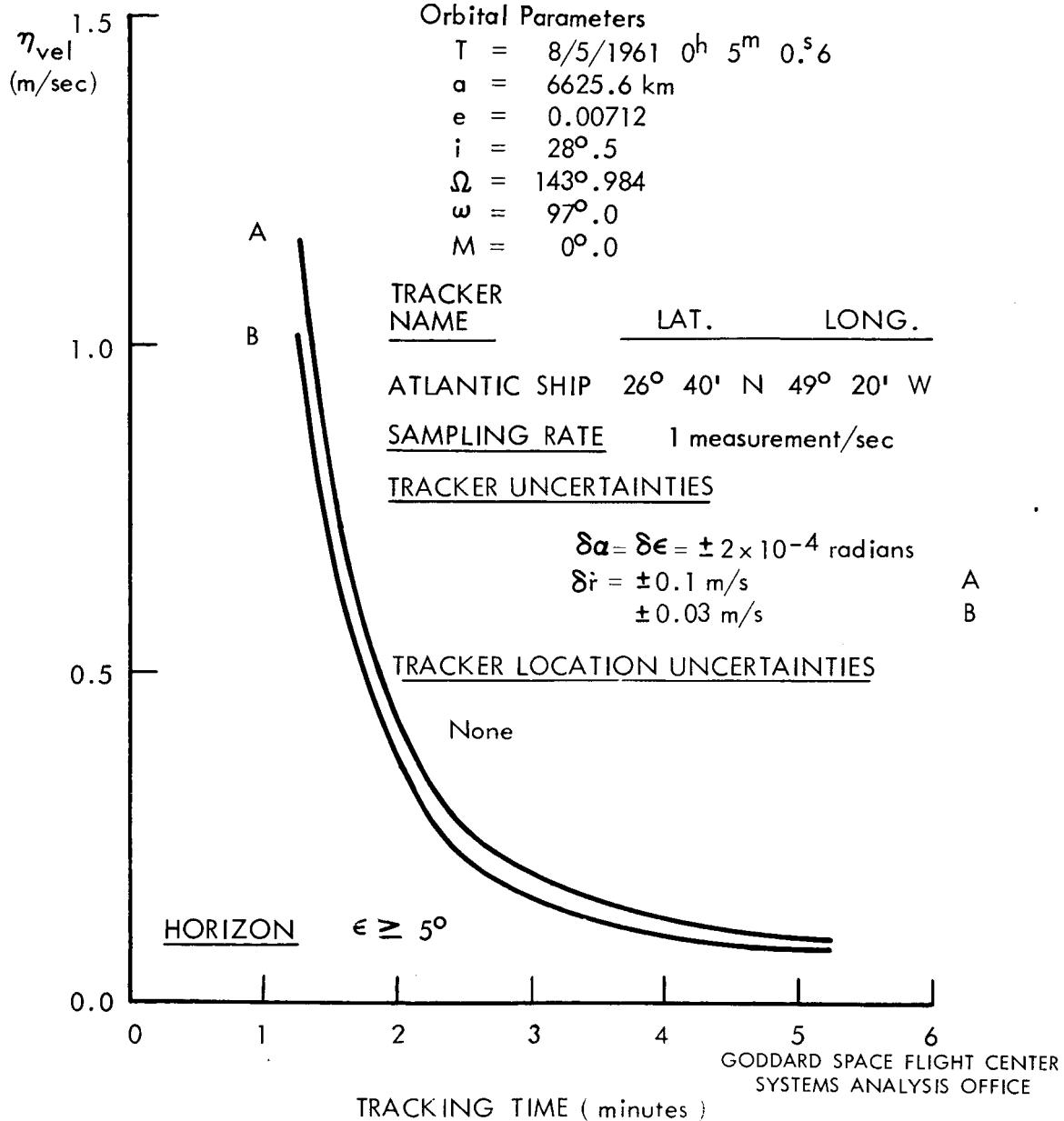


Figure 22

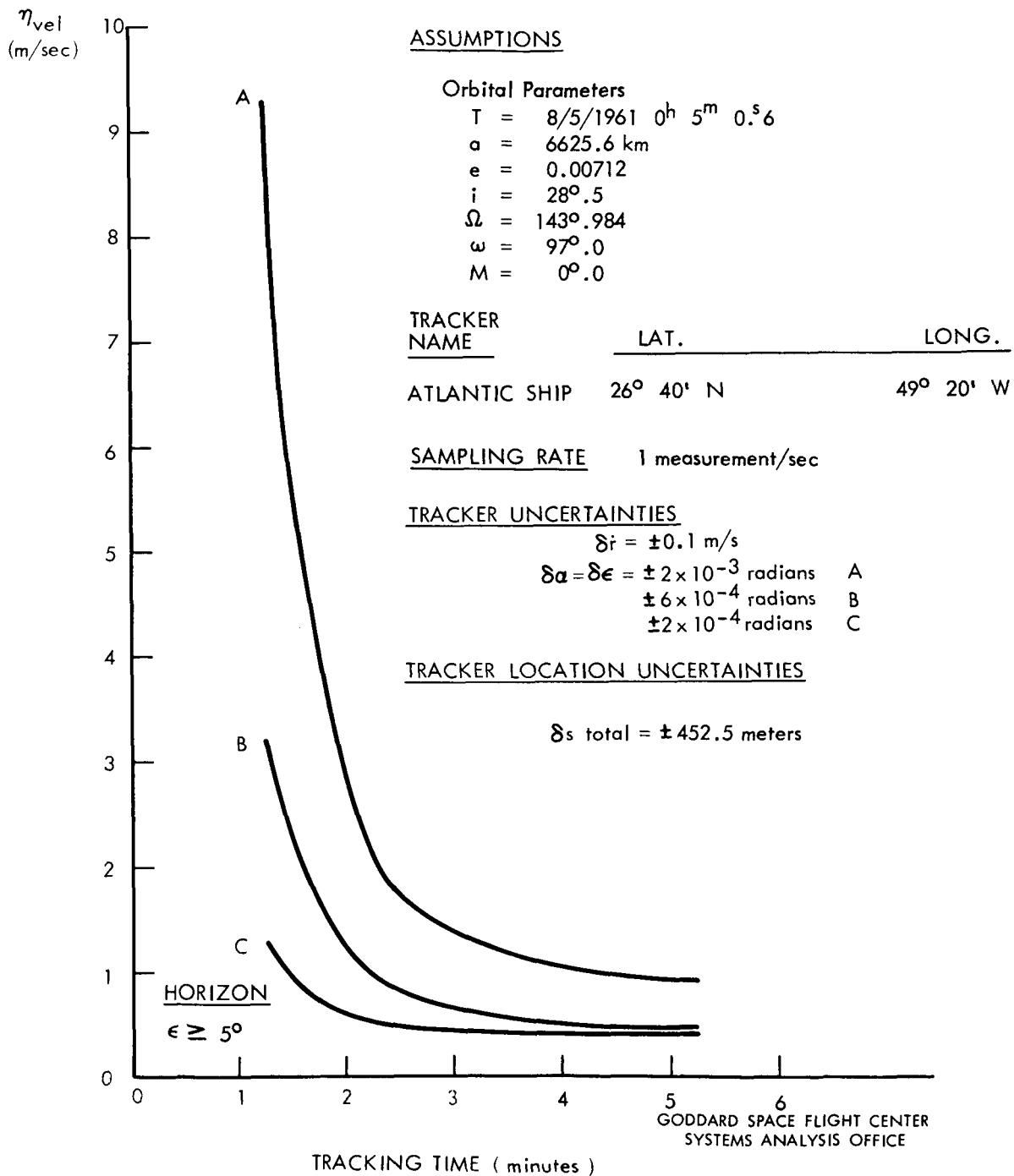


Figure 23

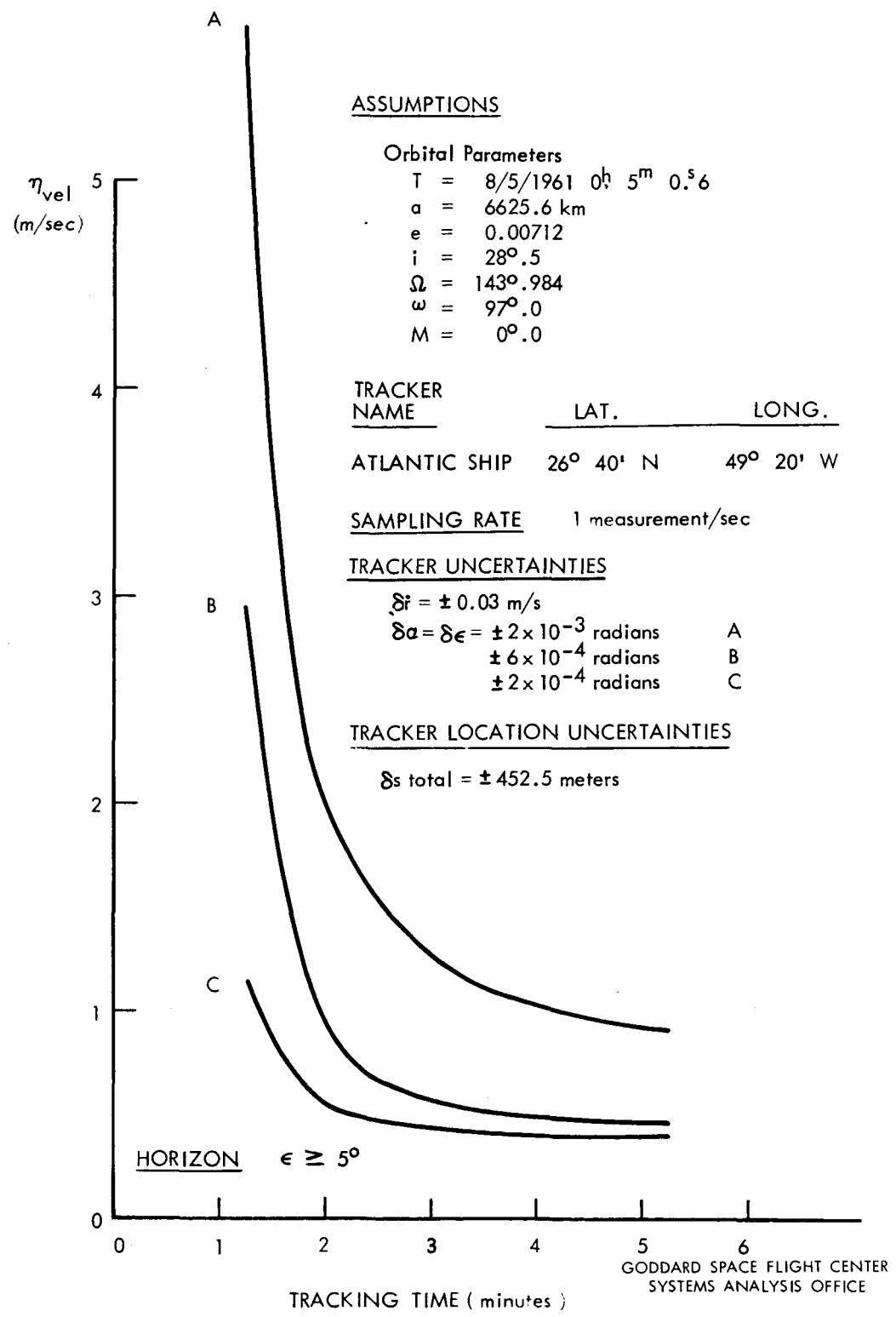


Figure 24

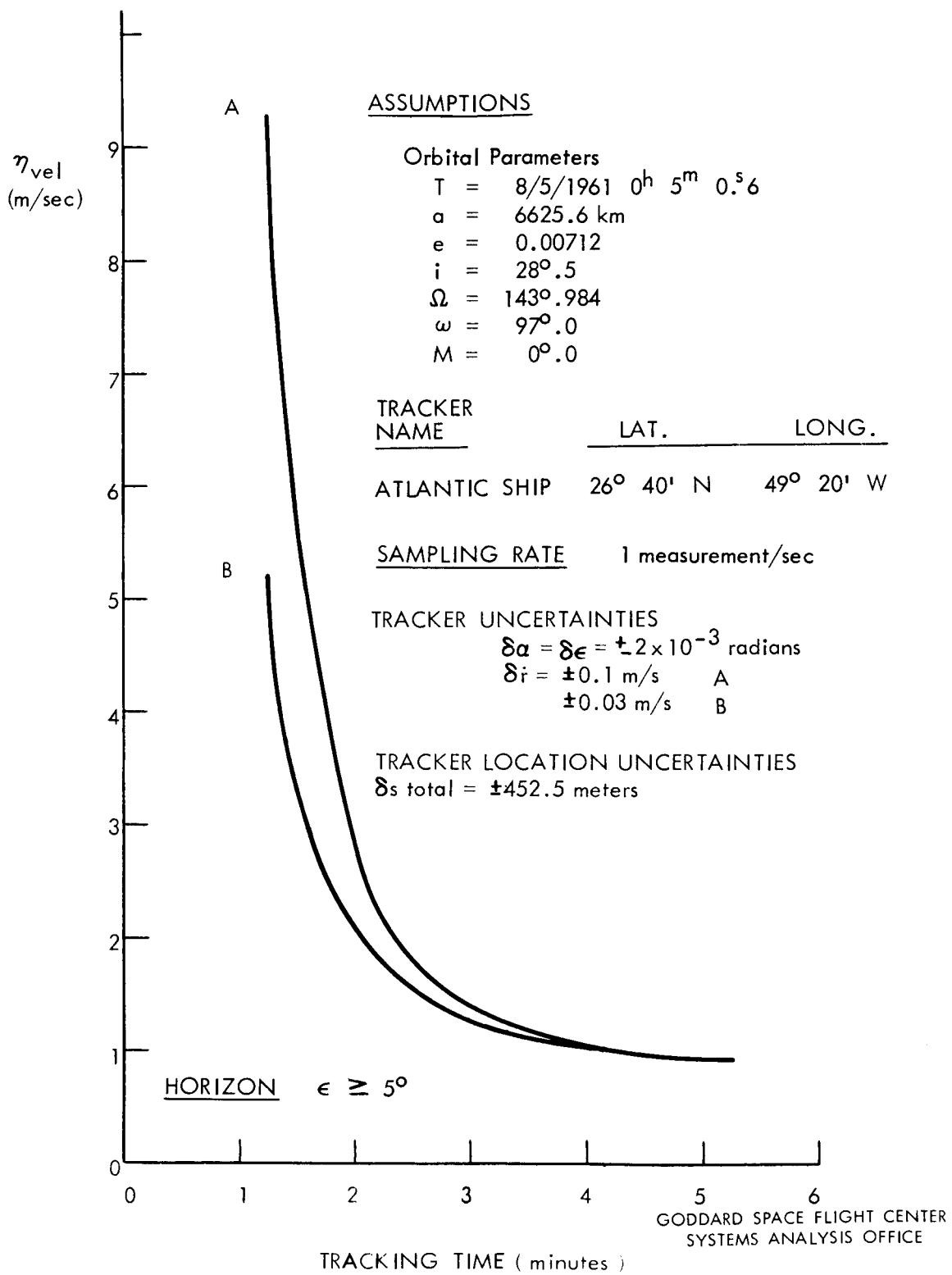


Figure 25

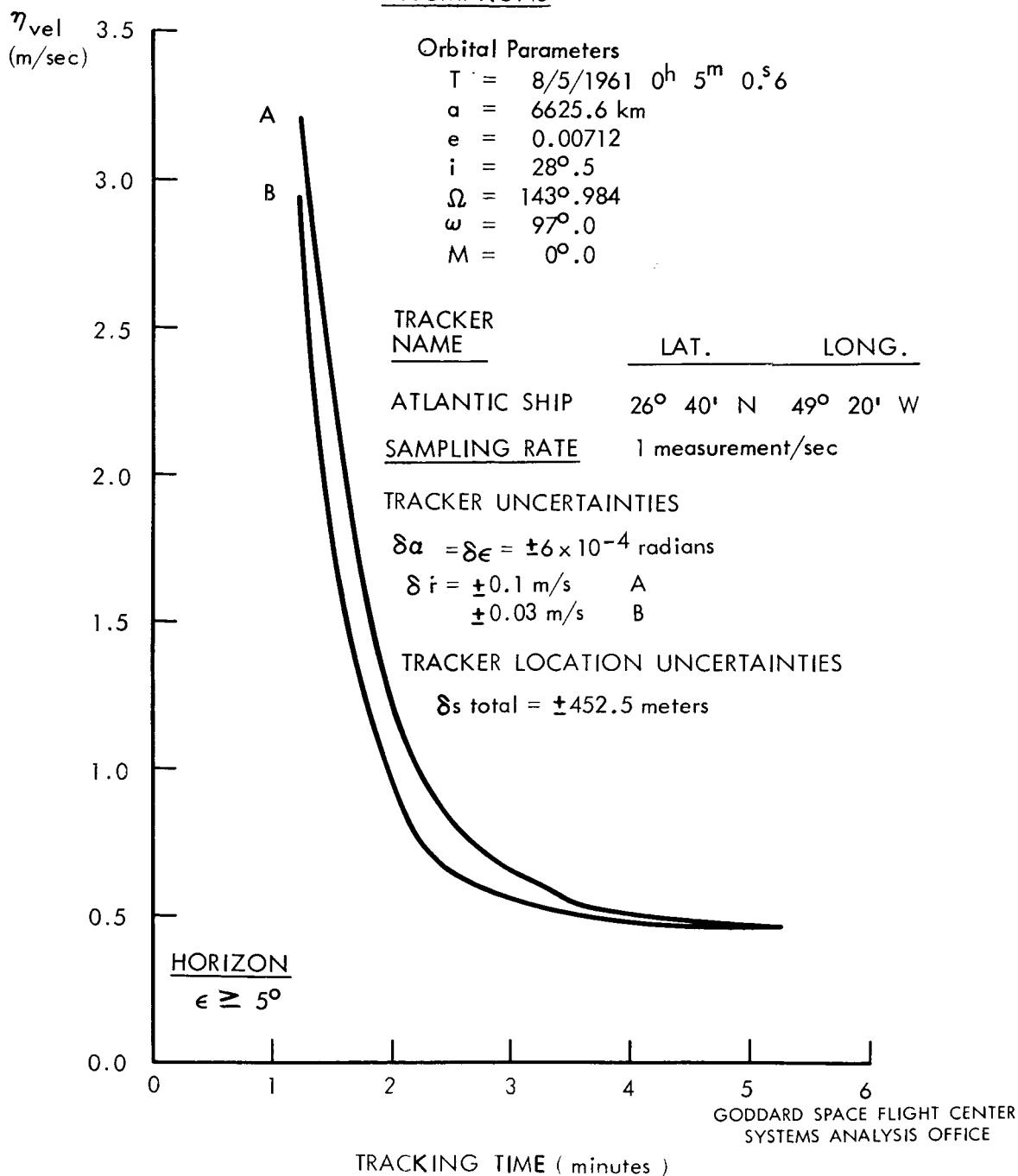


Figure 26

ASSUMPTIONS

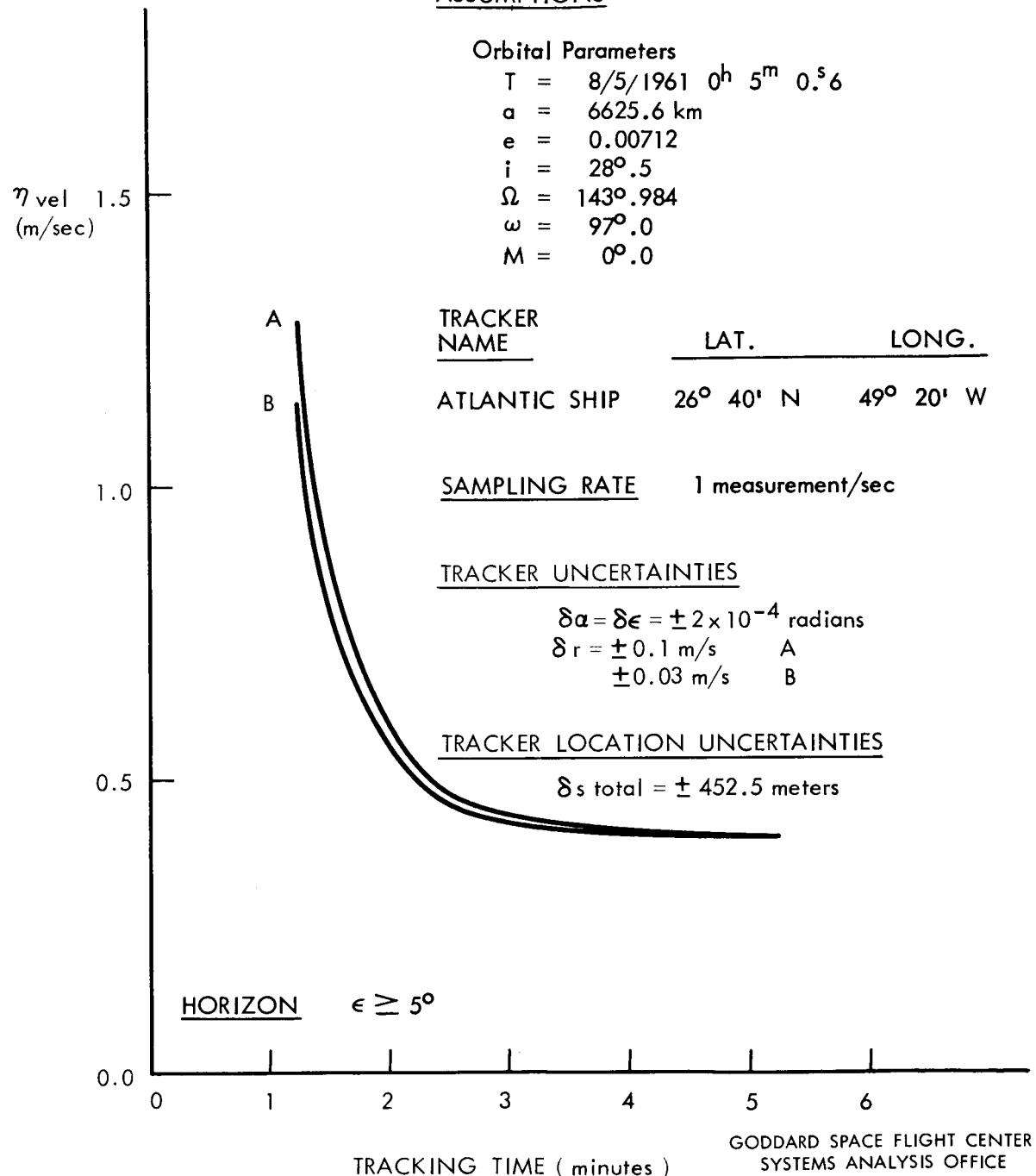


Figure 27

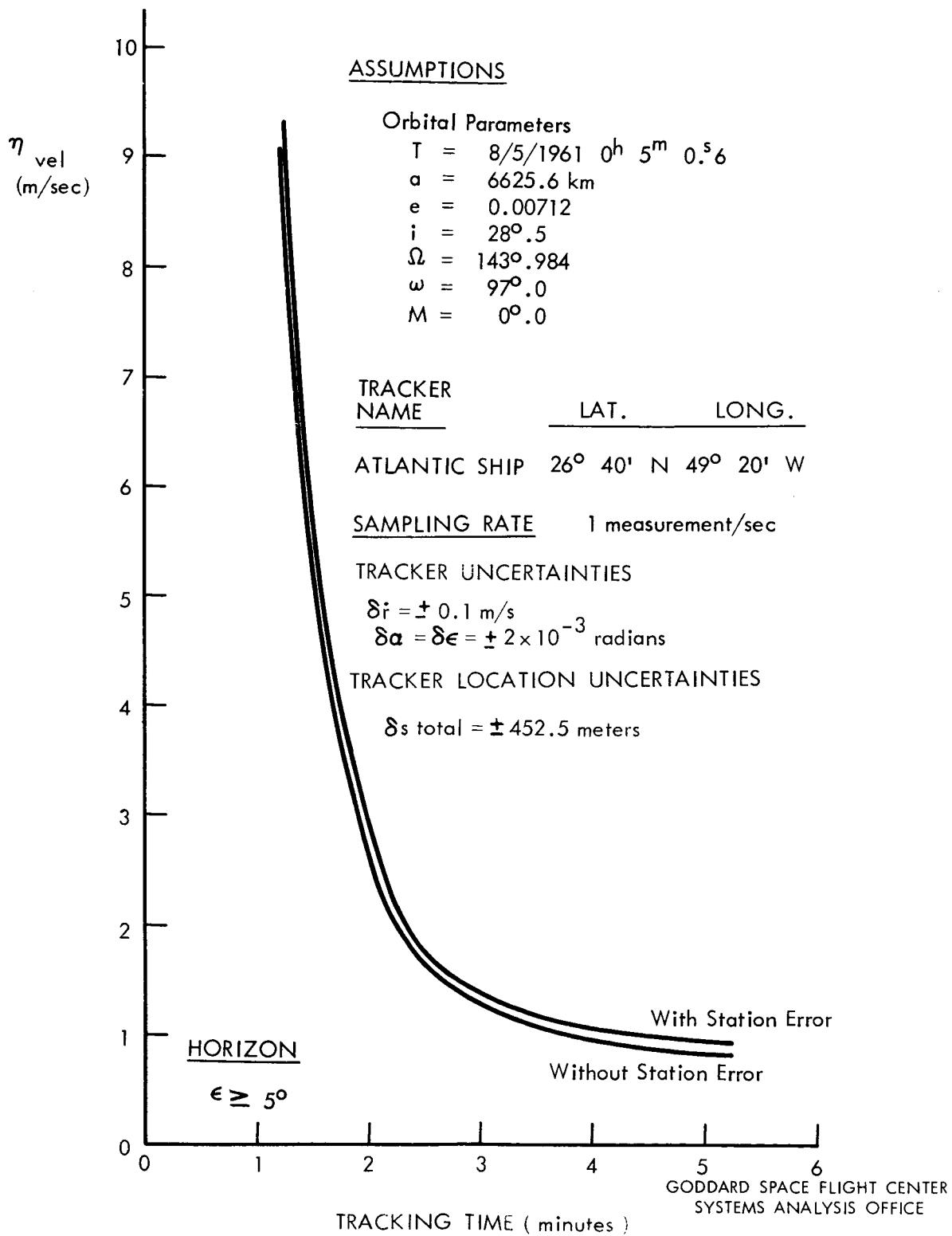


Figure 28

ASSUMPTIONS

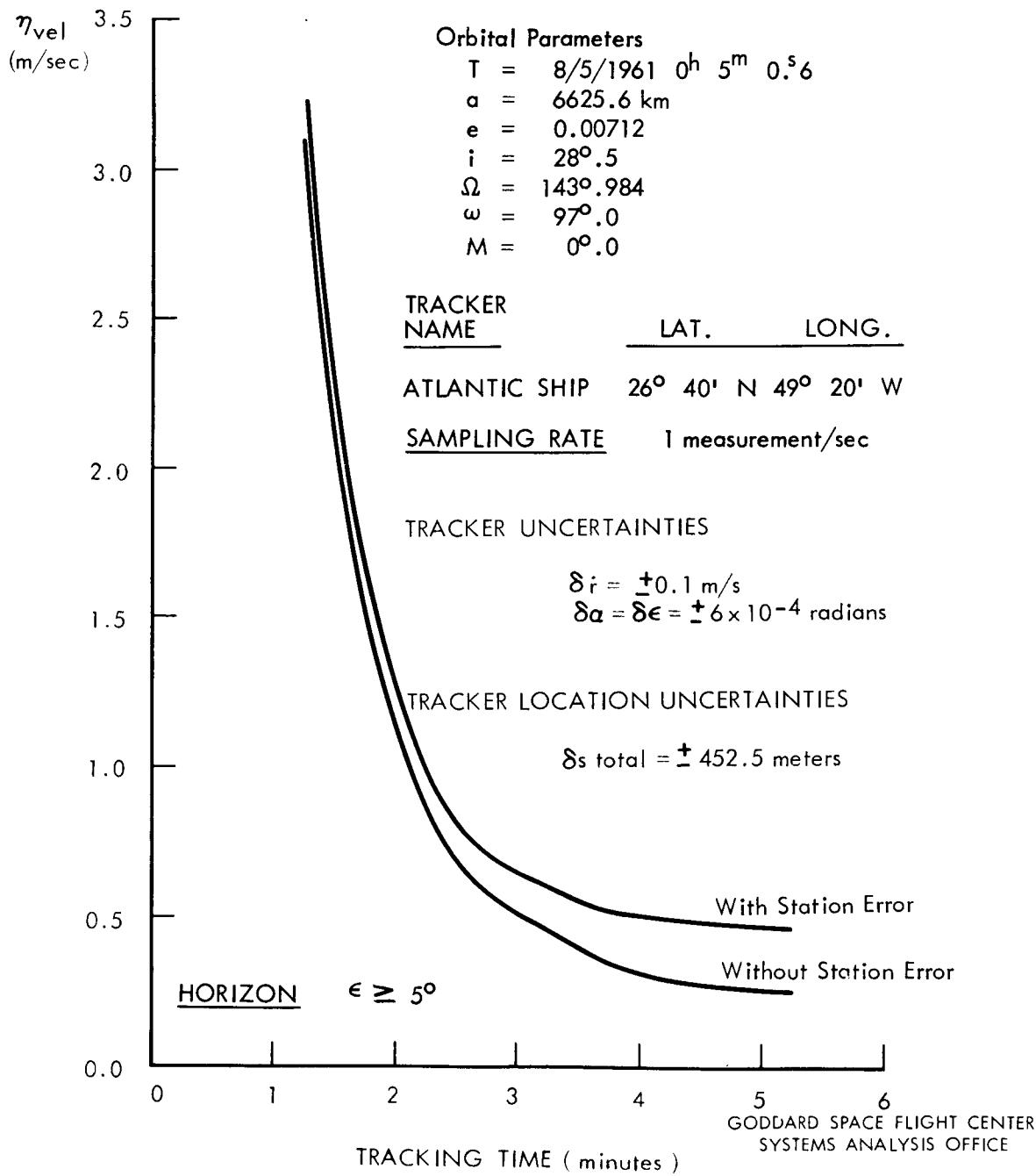


Figure 29

ASSUMPTIONS

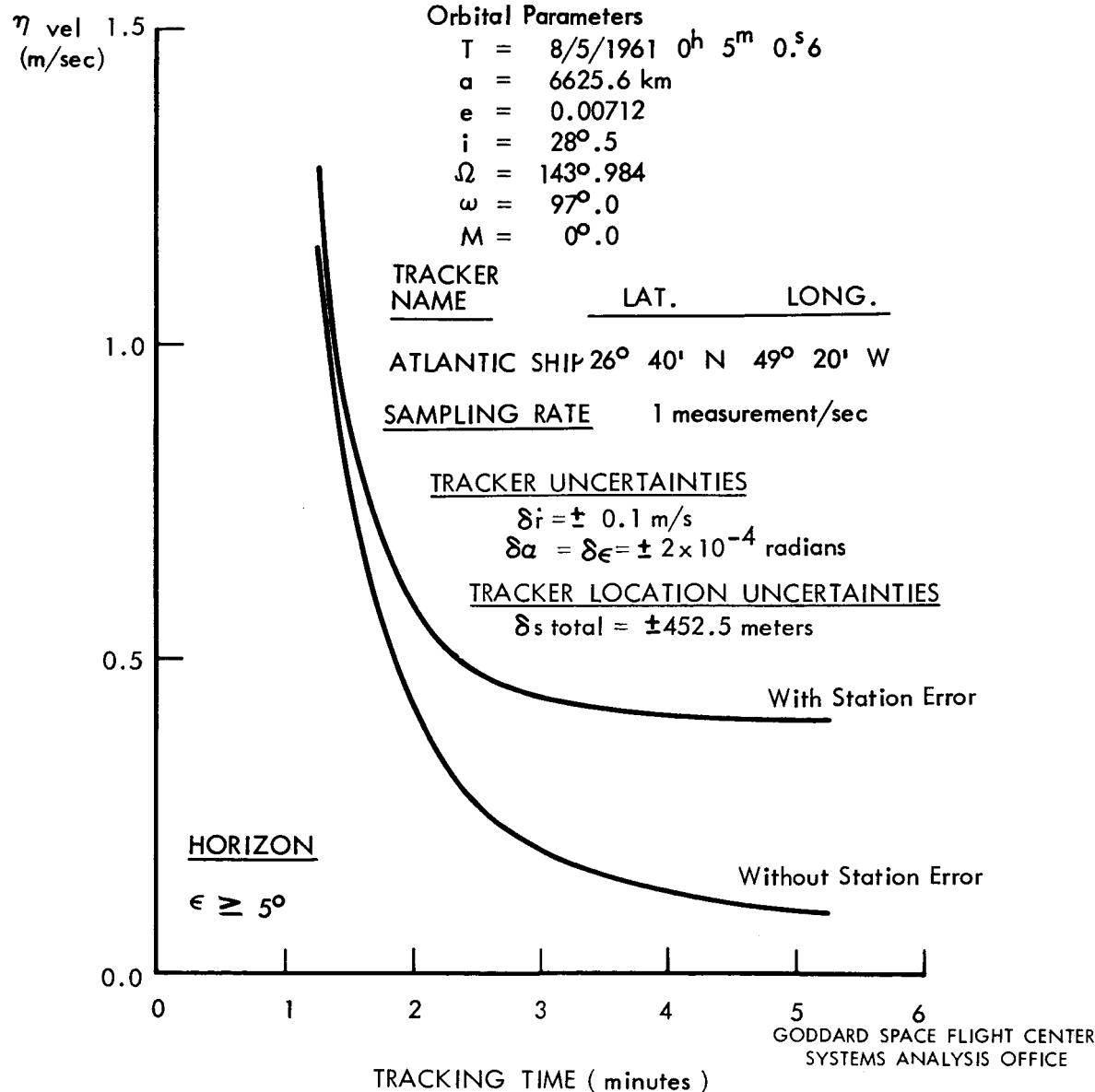


Figure 30

ASSUMPTIONS

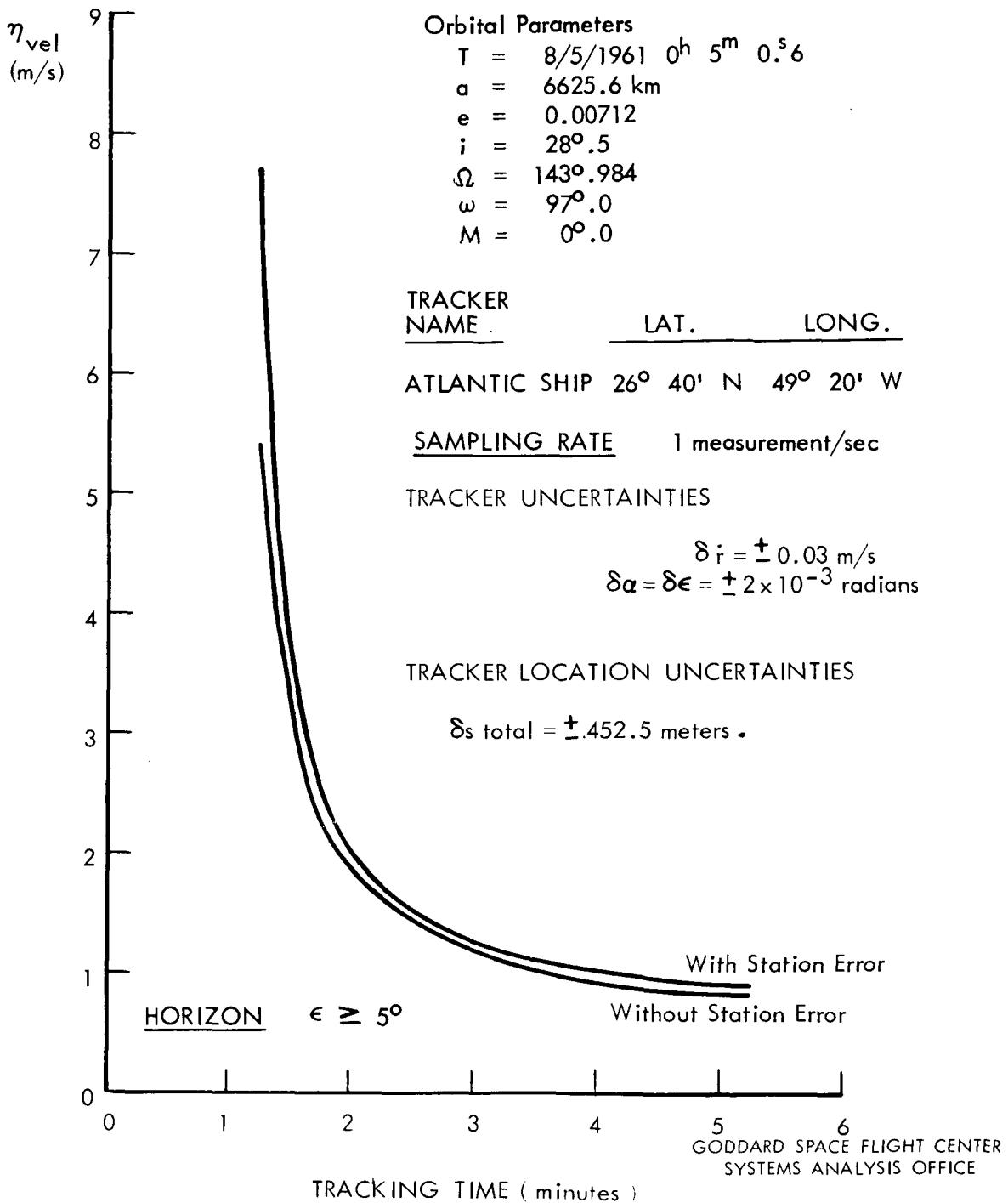


Figure 31

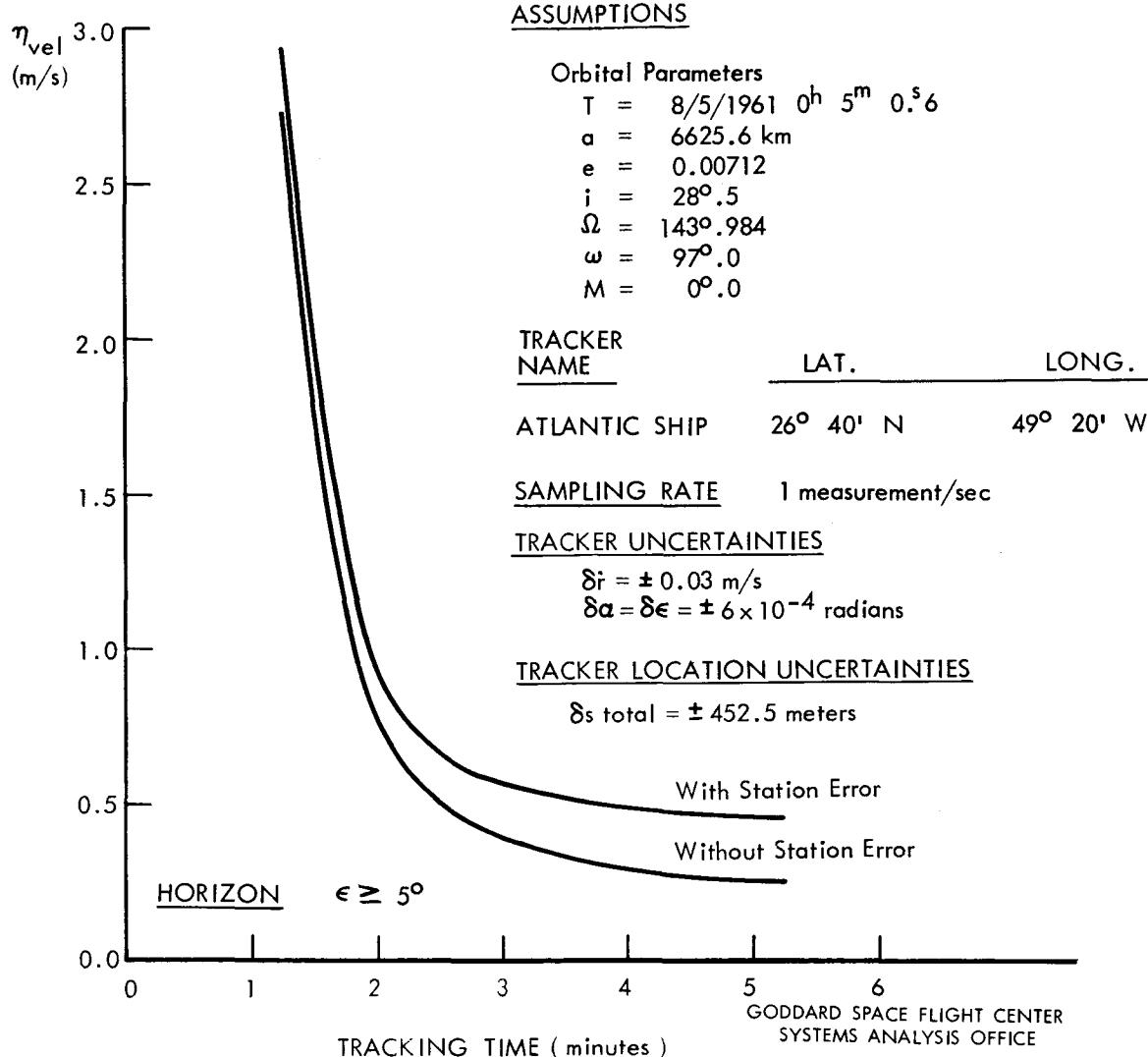


Figure 32

ASSUMPTIONS

Orbital Parameters

$T = 8/5/1961 \quad 0^h \ 5^m \ 0.^s6$
 $a = 6625.6 \text{ km}$
 $e = 0.00712$
 $i = 28^\circ.5$
 $\Omega = 143^\circ.984$
 $\omega = 97^\circ.0$
 $M = 0^\circ.0$

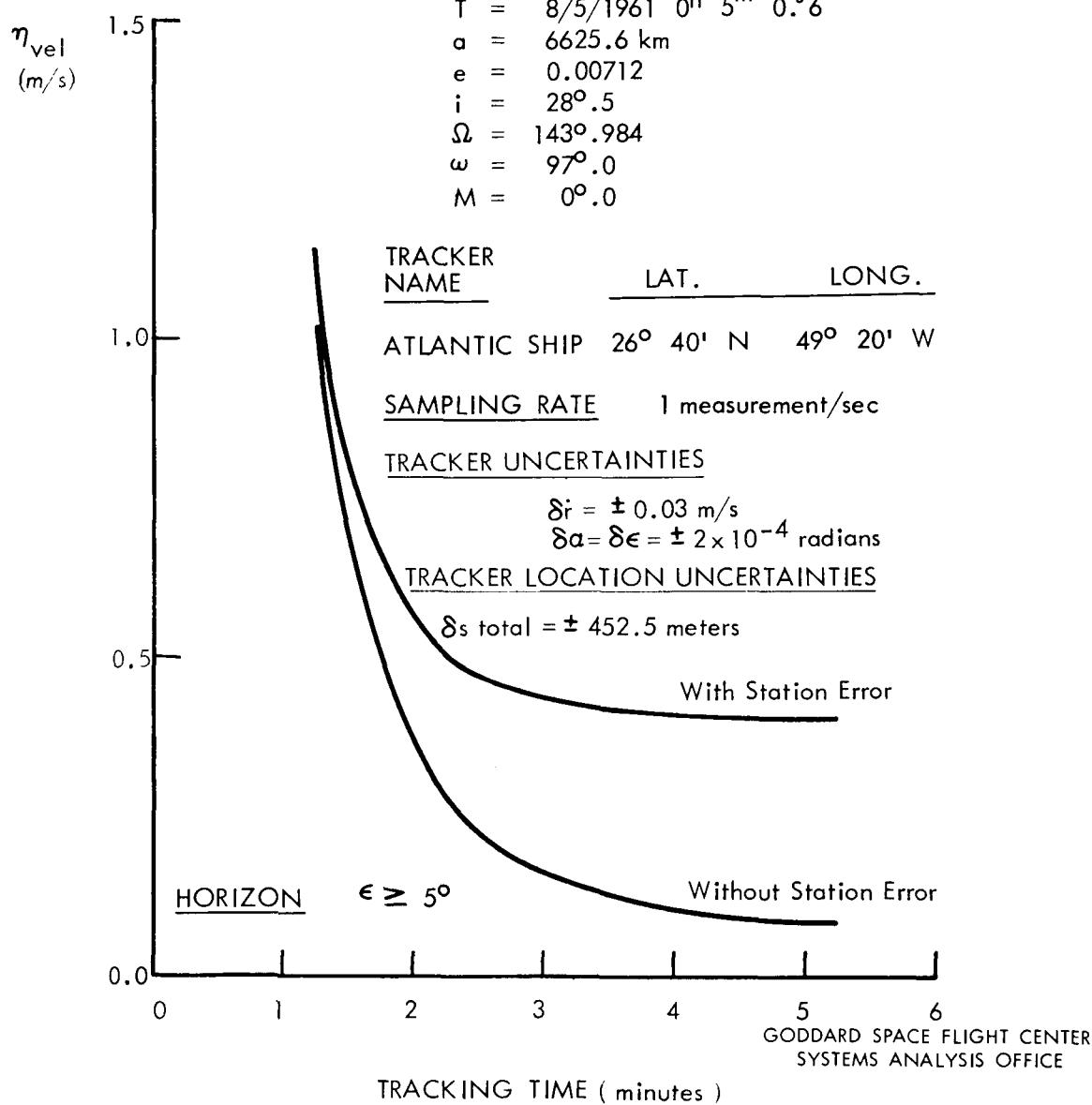


Figure 33